Name: ______________________________

Graphing Functions with Limited Domains

Today, we are going to graph pieces of a function by using equations with limited domains, meaning that they reach a point where the graph stops (on one or both ends) because *x* stops. To do this, we have to understand what the domain tells us, in terms of *x* and of how the point will look on the graph.

<u>When our domain (x) is between two limits</u>, the graph will have an **endpoint** (the graph will stop) on both sides.

- For our equation, we will plug in both of these *x*-values to find our two endpoints.
- If the equation is not a line ($y = mx + b$, for example), then we also plug in several numbers in between to find the correct shape of the graph.
- The endpoints that you find will either be an open point (∘) or a closed point (•), depending on whether or not the domain is "equal to" the limit value.
- Open points (∘) happen when the domain is less than/greater than, but **not equal to** the limit (< or >).
- Closed points (•) happen when the domain less than/greater than **or equal to** the limit (≤ or ≥)

For each domain below, identify the lower limit, upper limit, and whether each point will be open or closed.

Domain:	**Example** $-7 \le x \le 8$	**Example** $5 < x < 6$	**Example** $2 \le x < 4$	**Example** $-3 < x \le 1$
Lower Limit:	$x = -7$	$x = 5$	$x = 2$	$x = -3$
Open or Closed:	$-7 \le$ is CLOSED •	$5 <$ is OPEN ∘	$2 \le$ is CLOSED •	$-3 <$ is OPEN ∘
Upper Limit:	$x = 8$	$x = 6$	$x = 4$	$x = 1$
Open or Closed:	≤ 8 is CLOSED •	< 6 is OPEN ∘	< 4 is OPEN ∘	≤ 1 is CLOSED •

Domain:	1. $2 \le x < 8$	2. $-3 \le x \le -1$	**3.** $-6 < x \le 0$	**4.** $-8 < x < 1$
Lower Limit:				
Open or Closed:				
Upper Limit:				
Open or Closed:				

<u>When our domain (x) has only one limits</u>, the graph will only have an **endpoint** (the graph will stop) on one side.

- For our equation, we will plug in this *x*-values to find our only endpoint.
- Since we only have one point, we must pick another *x*-value(that does not go past the limit) to plug in.
- If the equation is not a line ($y = mx + b$, for example), then we will need to plug in several *x*'s to find the correct shape of the graph.
- When *x* is **greater than** the domain limit ($x >$ or $x \ge$), then we have a lower limit (because *x* is always more than that number), and the graph will continue **positively forever** (goes right until $+\infty$).
- When ***x* is less than** the domain limit ($x <$ or $x \le$), then we have an upper limit (because *x* is always less than that number), and the graph will continue **negatively forever** (goes left until $-\infty$).

Domain:	**Example** $x \ge -4$	**Example** $x > 6$	**Example** $x \le -7$	**Example** $x < -5$
Lower Limit:	$x = -4$ (x is bigger than -4)	$x = 6$ (x is bigger than 6)	*No lower limit, because* x gets smaller forever! $x = -\infty$	*No lower limit, because* x gets smaller forever! $x = -\infty$
Open or Closed:	≥ -4 is CLOSED •	> 6 is OPEN ∘		
Upper Limit:	*No upper limit, because* x gets bigger forever! $x = +\infty$	*No upper limit, because* x gets bigger forever! $x = +\infty$	$x = -7$ (x is smaller than -7)	$x = -5$ (x is smaller than -5)
Open or Closed:			≤ -7 is CLOSED •	< -5 is OPEN ∘

Domain:	5. $x < 0$	6. $x \ge -9$	7. $x > 8$	8. $x \le 4$
Lower Limit:				
Open or Closed:				
Upper Limit:				
Open or Closed:				

Now that we are able to understand and properly graph our domain limits as points, we can start practicing finding the endpoints and graphing pieces of an equation, limited by a domain.

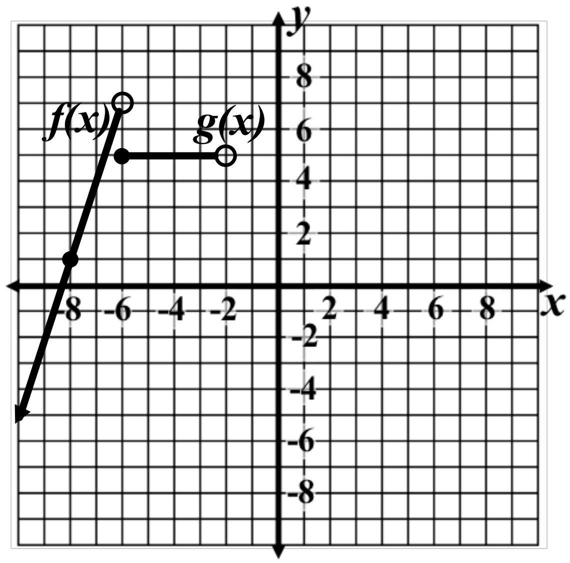
For each equation, plug in the *x*-values of the endpoints, determine if they will be open (∘) or closed (•), then fill in the *x*-*y* table. If necessary, plug in other *x*-values to determine the shape of the graph. Then, graph each equation piece on the given coordinate plane (graph problems 9 through 11 together with the examples).

Example
$f(x) = 3x + 25,$
$when\ x < -6$

	Lower limit ↓	Upper limit ↓
x	-8	-6 $(< -6 \to \circ)$
f(x)	1	7

Domain lower limit:
There's no lower limit, so I will pick any *x* that is less than -6. I'll use -8.
$f(-8) = 3(-8) + 25$
$f(-8) = -24 + 25$
$f(-8) = 1$
$(-8, 1)$

Domain upper limit:
$f(-6) = 3(-6) + 25$
$f(-6) = -18 + 25$
$f(-6) = 7$
$(-6, 7)$



Example
$g(x) = 5,$
$when - 6 \leq x < -2$

Same as:
$g(x) = 0x + 5$

	Lower limit ↓	Upper limit ↓
x	-6 $(\leq -6 \to \bullet)$	-2 $(< -2 \to \circ)$
g(x)	5	5

Domain lower limit:
$g(-6) = 0(-6) + 5$
$g(-6) = 5$
$(-6, 5)$

Domain upper limit:
$g(-2) = 0(-2) + 5$
$g(-2) = 5$
$(-2, 5)$

9. $h(x) = -2x - 1,$
$when - 2 \leq x < 0$

	Lower limit ↓	Upper limit ↓
x		
h(x)		

Domain lower limit:

Domain upper limit:

10. $j(x) = -3,$
$when\ 0 \leq x \leq 5$

	Lower limit ↓	Upper limit ↓
x		
h(x)		

Domain lower limit:

Domain upper limit:

11. $k(x) = x - 4,$
$when\ x > 5$

	Lower limit ↓	Upper limit ↓
x		
h(x)		

Domain lower limit:

Domain upper limit:

These five limited equations, when combined on one graph (like we did above) create something called a **Piecewise Function**.

The graph we created here could be written in piecewise format by combining all five equations and domains like this:

$$m(x) = \begin{cases} 3x + 25, & x < -6 \\ 5, & -6 \leq x < -2 \\ -2x - 1, & -2 \leq x < 0 \\ -3, & 0 < x \leq 5 \\ x - 4, & x > 5 \end{cases}$$