Name: _____

Probabilities with "And" (Part 2)

On the previous handout, we looked at "and" probabilities when each event is in its own category. Determining "and" probabilities out of one group, works the same way as it does in categories:

$$\text{P("and")} = (1st\ probability)(2nd\ probability)(3rd\ probability)(\dots)$$

An important aspect that you need to consider (whether it's in categories or not) is the effect that each event has on the events that come after. Will the first event change the sample space or sample size for the second event?

The proper way to write "probability of the 2nd event given that the 1st event happened" is: $P(2nd\ event|1st\ event)$. The "|" means "given that" _____ happens or "after" _____ happens.

"And" Using One Group: With or Without Replacement

EXAMPLE There are 9 shirts, 4 sweatshirts and 5 pairs of shoes in the closet. If you randomly select 3 items (without putting any of them back), what is the sample size for the combined probability? What is the probability that you will select a shirt, a sweatshirt and a pair of shoes, if none of the items are replaced?

Event	1st item – Want a shirt			2nd item – Want a sweatshirt (After getting a shirt)			3rd item – Want shoes (After getting shirt & sweatshirt)		
Sample Space	Shirt	Sweatshirt	Shoes	Shirt	Sweatshirt	Shoes	Shirt	Sweatshirt	Shoes
Amount	9	4	5	8 are left	4	5	8 are left	3 are left	5
Sample Size	18 total	18 total	18 total	17 total are left	17 total are left	17 total are left	16 total are left	16 total are left	16 total are left
Probability of each event	$\frac{9}{18}=\frac{1}{2}$	$\frac{4}{18}=\frac{2}{9}$	$\frac{5}{18}$	$\frac{8}{17}$	$\frac{4}{17}$	$\frac{5}{17}$	$\frac{8}{16}=\frac{1}{2}$	$\frac{3}{16}$	$\frac{5}{16}$

a. Combined Sample Size: 1st event: 18 total 2nd event: 17 total are left 3rd event: 16 total are left

$$Combined\ Sample\ Size = (18)(17)(16) = (306)(16) = \boxed{4{,}896}$$

b. $P(Shirt\ \&\ Sweatshirt\ \&\ Shoes) = P(Shirt) \cdot P(Sweatshirt|Shirt) \cdot P(Shoes|Shirt\ \&\ Sweatshirt)$

$$P(Shirt\ \&\ Sweatshirt\ \&\ Shoes) = \left(\frac{1}{2}\right)\left(\frac{4}{17}\right)\left(\frac{5}{16}\right) = \frac{(1)(4)(5)}{(2)(17)(16)} = \frac{20}{544} \dots reduce\ by\ 4 \dots \boxed{\frac{5}{136}}$$

EXAMPLE There are 9 shirts, 4 sweatshirts and 5 pairs of shoes in the closet. If you randomly select 3 items (and put all of them back), what is the sample size for the combined probability? What is the probability that you will select a shirt, a sweatshirt and a pair of shoes, if all of the items are replaced?

Event	1st item – Want a shirt			2nd item – Want a sweatshirt (After putting a shirt back)			3rd item – Want shoes (After putting shirt & sweatshirt back)		
Sample Space	Shirt	Sweatshirt	Shoes	Shirt	Sweatshirt	Shoes	Shirt	Sweatshirt	Shoes
Amount	9	4	5	Still 9	4	5	Still 9	Still 4	5
Sample Size	18 total	18 total	18 total	Still 18	Still 18	Still 18	Still 18	Still 18	Still 18
Probability of each event	$\frac{9}{18}=\frac{1}{2}$	$\frac{4}{18}=\frac{2}{9}$	$\frac{5}{18}$	$\frac{9}{18}=\frac{1}{2}$	$\frac{4}{18}=\frac{2}{9}$	$\frac{5}{18}$	$\frac{9}{18}=\frac{1}{2}$	$\frac{4}{18}=\frac{2}{9}$	$\frac{5}{18}$

a. Combined Sample Size: 1st event: 18 total 2nd event: 18 is still the total 3rd event: 18 is still the total

$$Combined\ Sample\ Size = (18)(18)(18) = (324)(18) = \boxed{5{,}832}$$

b. $P(Shirt\ \&\ Sweatshirt\ \&\ Shoes) = P(Shirt) \cdot P(Sweatshirt|Replaced\ Shirt) \cdot P(Shoes|Replaced\ Shirt\ \&\ Sweatshirt)$

$$P(Shirt\ \&\ Sweatshirt\ \&\ Shoes) = \left(\frac{1}{2}\right)\left(\frac{2}{9}\right)\left(\frac{5}{18}\right) = \frac{(1)(2)(5)}{(2)(9)(18)} = \frac{10}{324} \dots reduce\ by\ 4 \dots \boxed{\frac{5}{162}}$$

1. There are 8 guys and 8 girls running for one of the following officer positions: club treasurer, vice president, and president. If you randomly select a person for each position, what is the sample size for the combined probability? What is the probability of selecting a male treasurer, female vice president, and male president? *(One thing to consider: Can the same person be the president, the vice president and the treasurer AT THE SAME TIME?)*

Event	1st item – Want a Male Tr.		2nd item – Want a Female VP (After taking out a Male Tr.)		3rd item – Want a Male Pres. (After taking out a M Tr. & F VP)	
Sample Space	Male	Female	Male	Female	Male	Female
Amount	8	8	___ males left	8	___ males left	__ females left
Sample Size	8	8	___ total left	___ total left	___ total left	___ total left
Probability						

a. Combined Sample Size:

b. $P(Male\ Tr. \,\&\, Female\ VP \,\&\, Male\ Pres.) = P(Male\ Tr.) \cdot P(Female\ VP|MT)\ \&P(Male\ Pres.|MT\ \&\ FVP)$

2. There are 6 freshmen and 10 sophomores who have their names in a raffle. If 4 names are randomly selected, and then put back into the raffle, what is the sample size for the combined probability? What is the probability that all of the four winners will be sophomores? *(If the names are put back in the raffle, is it possible for 1 person win all 4 times?)*

Category	1st winner – Want Soph		2nd winner – Want Soph (After put back soph)		3rd winner – Want Soph (After put back 2 soph)		4th winner – Want Soph (After put back 2 soph)	
Sample Space	Fresh.	Soph.	Fresh.	Soph.	Fresh.	Soph.	Fresh.	Soph.
Amount								
Sample Size								
Probability								

a. Combined Sample Size:

b. $P(Soph\ \&\ Soph\ \&\ Soph\ \&\ Soph) =$

3. There are 6 freshmen and 10 sophomores who have their names in a raffle. If 4 names are randomly selected, and their names are NOT put back into the raffle, what is the sample size for the combined probability? What is the probability that all of the four winners will be sophomores? *(If the names are not put back in the raffle, is it possible for 1 person win all 4 times?)*

Category	1st winner – Want Soph		2nd winner – Want Soph (After took out soph)		3rd winner – Want Soph (After took out 2 soph)		4th winner – Want Soph (After took out 2 soph)	
Sample Space								
Amount								
Sample Size								
Probability								

a. Combined Sample Size:

b. $P(Soph\ \&\ Soph\ \&\ Soph\ \&\ Soph) =$

4. There are 12 baseballs & 3 tennis balls, 5 baseball jerseys, 3 football jerseys, and 2 plain shirts in the a bag. If you randomly select 2 items from the bag, what is the sample size for the combined probability without replacement? What is the probability of selecting a tennis ball and a plain shirt without replacement? *(Without replacement means they did not put them back. Will that affect the future events?)*

Category										
Sample Space										
Amount										
Sample Size										
Probability of each event										

a. Combined Sample Size:

b. $P(Tennis\ Ball\ \&\ Plain\ Shirt) =$

5. There are 12 baseballs & 3 tennis balls, 5 baseball jerseys, 3 football jerseys, and 2 plain shirts in the a bag. If you randomly select 2 items from the bag, what is the sample size for the combined probability with replacement? What is the probability of selecting a tennis ball and a plain shirt with replacement? *(With replacement means they put them back. Will that affect the future events?)*

Category										
Sample Space										
Amount										
Sample Size										
Probability of each event										

a. Combined Sample Size:

b. $P(Tennis\ Ball\ \&\ Plain\ Shirt) =$

6. There are 3 blue shirts, 2 gray shirts, 1 black shirt, and 2 pairs of jeans in a drawer. If you randomly select 3 items with replacement, what is the sample size for the combined probability? What is the probability of selecting a gray shirt, a pair of jeans, and black shirt? *(With replacement means they put them back. Will that affect the future events?)*

a. Combined Sample Size:

b. $P(Gray\ Shirt\ \&\ Jeans\ \&\ Black\ Shirt) =$

7. There are 3 blue shirts, 2 gray shirts, 1 black shirt, and 2 pairs of jeans in a drawer. If you randomly select 3 items without replacement, what is the sample size for the combined probability? What is the probability of selecting a gray shirt, a pair of jeans, and black shirt? *(Without replacement means they did not put them back. Will that affect the future events?)*

a. Combined Sample Size:

b. $P(Gray\ Shirt\ \&\ Jeans\ \&\ Black\ Shirt) =$

Basics of Probability with "And" (Part 2)
Answers

1. $Combined\ Sample\ Space = \boxed{3{,}360}$ $P(Male\ Tr.\ \&\ Female\ VP\ \&\ Male\ Pres.) = \boxed{\frac{2}{15}}$	2. $Combined\ Sample\ Space = \boxed{65{,}536}$ $P(Soph\ \&\ Soph\ \&\ Soph\ \&\ Soph) =.) = \boxed{\frac{625}{4096}}$
3. $Combined\ Sample\ Space = \boxed{43{,}680}$ $P(Soph\ \&\ Soph\ \&\ Soph\ \&\ Soph) = \boxed{\frac{3}{26}}$	4. $Combined\ Sample\ Space = \boxed{600}$ $P(Tennis\ Ball\ \&\ Plain\ Shirt) = \boxed{\frac{1}{100}}$
5. $Combined\ Sample\ Space = \boxed{625}$ $P(Tennis\ Ball\ \&\ Plain\ Shirt) = \boxed{\frac{6}{625}}$	6. $Combined\ Sample\ Space = \boxed{512}$ $P(Gray\ Shirt\ \&\ Jeans\ \&\ Black\ Shirt) = \boxed{\frac{1}{128}}$
7. $Combined\ Sample\ Space = \boxed{336}$ $P(Gray\ Shirt\ \&\ Jeans\ \&\ Black\ Shirt) = \boxed{\frac{1}{84}}$	