

Name: _____

Probability Using Two-way Frequency Tables

A two-way frequency table is a way of representing data that fits into multiple categories. Any probability problem that has more than one overlapping category can be re-written as a two-way frequency table.

Example:

“If there are 8 junior baseball players, 4 junior soccer players, 7 senior baseball players, 5 senior track & field athletes, and 6 senior soccer players...”

To set up my table, I start with my two categories (which are called “variables”): their grade level and their sport, (including a row and a column to write in the totals). Then, I fill in the information that I know.

	Grade		
	Junior	Senior	TOTAL
Sport	Baseball		
	Soccer		
	Track & Field		
	TOTAL		

→

	Grade		
	Junior	Senior	TOTAL
Sport	Baseball	JUNIOR BASEBALL	SENIOR BASEBALL
	Soccer	JUNIOR SOCCER	SENIOR SOCCER
	Track & Field	JUNIOR TRACK & FIELD	SENIOR TRACK & FIELD
	TOTAL		

	Grade		
	Junior	Senior	TOTAL
Sport	Baseball	8	7
	Soccer	4	6
	Track & Field	0	5
	TOTAL	12 Junior	18 Senior

So, according to the table, the probability of randomly selecting a **junior baseball player** would be:

$$P(\text{junior baseball player}) = \frac{8 \text{ Jr BB}}{30 \text{ Total}} = \frac{4}{15}$$

If I wanted the probability that he was a junior OR a baseball player, I would use count up those categories.

	Grade		
	Junior	Senior	TOTAL
Sport	Baseball	8	7
	Soccer	4	6
	Track & Field	0	5
	TOTAL	12 Junior	18 Senior

Acceptable outcomes:

8 Jr. baseball,
7 Sr. baseball
4 Jr. Soccer
& 0 Jr. Track & Field
= 19 Total

$$P(\text{junior OR baseball player}) = \frac{19}{30}$$

I could also have done
 $15 \text{ baseball} + 12 \text{ junior} - 8 \text{ both} = 27 - 8 = 19$

I could also find the probability of randomly selecting a baseball player given that he is a junior (“given that” means he has to be a junior). For this probability, I would ignore all options that are not juniors:

$$P(\text{Baseball} | \text{Junior}) = \frac{8 \text{ BB}}{12 \text{ Jr}} = \frac{2}{3}$$

EXAMPLE

		Color		TOTAL
		Black	Blue	
Clothing Item	Shirts	12	8	20
	Jackets	3	2	5
	Pants	9	6	15
	TOTAL	24	16	40

a. Name the two variables displayed in the table.
Color & Clothing

b. If an item is selected at random, what is the probability that it is a jacket?

$$P(\text{blue jacket}) = \frac{\text{total jackets}}{\text{overall total}} = \frac{5}{40} = \frac{1}{8}$$

c. What is the probability that a randomly selected item is black or a shirt?

The items that meet the requirements are:

12 **black shirts**, 3 **black jackets**, 9 **black pants**, and 8 **blue shirts** = 12 + 3 + 9 + 8 = 32

$$P(\text{black or shirt}) = \frac{32}{40} = \frac{4}{5}$$

d. What is the probability that a randomly selected item is a pair of pants given that the item is blue?

$$P(\text{pants}|\text{blue}) = \frac{\text{pants in blue category}}{\text{blue total}} = \frac{6}{16} = \frac{3}{8}$$

1.

		Employment		TOTAL
		Has a job	Does not have a job	
Gender	Male	27	36	63
	Female	28	24	52
	TOTAL	55	60	115

a. Name the two variables displayed in the table.

b. If a person is selected at random, what is the probability that he or she has a job?

c. What is the probability that a randomly selected person is male or has a job?

d. What is the probability that a randomly selected person is female given that the person has a job?

EXAMPLE

		Grade				TOTAL
		9 th	10 th	11 th	12 th	
Club Participation	Is in a club	105	125	147	101	478
	Is not in a club	78	92	75	122	367
	TOTAL	183	217	222	223	845

a. Name the two variables displayed in the table.
Grade & Club Participation

b. If a student is selected at random, what is the probability that he or she will be in a club?

$$P(\text{in a club}) = \frac{\text{total in clubs}}{\text{overall total}} = \frac{478}{845}$$

c. What is the probability that a randomly selected student is in a club or in 11th grade?

The students that meet the requirements are:

105 **9th in clubs**, 125 **10th in clubs**, 147 **11th in clubs**, 101 **12th in clubs** and 75 **11th not in clubs**
= 105 + 125 + 147 + 101 + 75 = 553

$$P(\text{in clubs or 11th}) = \frac{553}{845}$$

d. What is the probability that a randomly selected student is in a club given that he or she is in 10th?

$$P(\text{in club}|10th) = \frac{\text{in club in 10th}}{10th \text{ total}} = \frac{125}{217}$$

2.

		Color			TOTAL
		Yellow	Pink	Silver	
Item	Post-it	58	17	0	75
	Paper Clip	7	25	78	110
	TOTAL	65	42	78	185

a. Name the two variables displayed in the table.

b. If an item is selected at random, what is the probability that it is a paper clip?

c. What is the probability that a randomly selected item is pink or a post-it?

d. What is the probability that a randomly selected item is silver given that it's a paper clip?

<p>3.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="2"></th> <th colspan="4">Grade</th> <th></th> </tr> <tr> <th colspan="2"></th> <th>9th</th> <th>10th</th> <th>11th</th> <th>12th</th> <th>TOTAL</th> </tr> </thead> <tbody> <tr> <th rowspan="3" style="writing-mode: vertical-rl; transform: rotate(180deg);">Gender</th> <th>Male</th> <td>204</td> <td>179</td> <td>165</td> <td>202</td> <td>750</td> </tr> <tr> <th>Female</th> <td>170</td> <td>246</td> <td>143</td> <td>131</td> <td>690</td> </tr> <tr> <th>TOTAL</th> <td>374</td> <td>425</td> <td>308</td> <td>333</td> <td>1440</td> </tr> </tbody> </table> <p>a. Name the two variables displayed in the table.</p> <p>b. If a student is selected at random, what is the probability that he or she will be in 9th grade?</p> <p>c. What is the probability that a randomly selected student is in 10th grade or female?</p> <p>d. What is the probability that a randomly selected student is in 11th grade given that the student is female?</p>			Grade							9 th	10 th	11 th	12 th	TOTAL	Gender	Male	204	179	165	202	750	Female	170	246	143	131	690	TOTAL	374	425	308	333	1440	<p>4.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th colspan="2"></th> <th colspan="3">Sports Participation</th> <th></th> </tr> <tr> <th colspan="2"></th> <th>Tennis</th> <th>Soccer</th> <th>Not in Sports</th> <th>TOTAL</th> </tr> </thead> <tbody> <tr> <th rowspan="3" style="writing-mode: vertical-rl; transform: rotate(180deg);">Club Participation</th> <th>Is in a club</th> <td>145</td> <td>106</td> <td>138</td> <td>389</td> </tr> <tr> <th>Is not in a club</th> <td>123</td> <td>164</td> <td>108</td> <td>395</td> </tr> <tr> <th>TOTAL</th> <td>268</td> <td>270</td> <td>246</td> <td>784</td> </tr> </tbody> </table> <p>a. Name the two variables displayed in the table.</p> <p>b. If a person is selected at random, what is the probability that he or she plays tennis?</p> <p>c. What is the probability that a randomly selected student plays soccer or is not in a club?</p> <p>d. What is the probability that a randomly selected student is not in a club given that he or she does not play sports?</p>			Sports Participation						Tennis	Soccer	Not in Sports	TOTAL	Club Participation	Is in a club	145	106	138	389	Is not in a club	123	164	108	395	TOTAL	268	270	246	784
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“Probability Using Two-way Frequency Tables” Answers

<p>1a. Employment & Gender</p> <p>1b. $\frac{11}{23}$ 1c. $\frac{91}{115}$ 1d. $\frac{28}{55}$</p>	<p>2a. Color & Item</p> <p>2b. $\frac{22}{37}$ 2c. $\frac{20}{37}$ 2d. 1</p>	<p>3a. Grade & Gender</p> <p>3b. $\frac{187}{720}$ 3c. $\frac{869}{1440}$ 3d. $\frac{143}{690}$</p>
<p>4a. Sports & Club Participation</p> <p>4b. $\frac{67}{196}$ 4c. $\frac{501}{784}$ 4d. $\frac{16}{41}$</p>	<p>5a. Employment & Age</p> <p>5b. $\frac{179}{306}$ 5c. $\frac{73}{102}$ 5d. $\frac{211}{259}$</p>	<p>6a. Phone Preference & Grade</p> <p>6b. $\frac{131}{258}$ 6c. $\frac{1419}{2322} = \frac{11}{18}$ 6d. $\frac{143}{281}$</p>