

Details of a Quadratic from the Equations & the Graph

EXAMPLE 1. Standard form: $f(x) = x^2 - 6x + 8$

$f(x) = x^2 - 6x + 8$

y-int: (0, 8) or y = 8

Factored form: $f(x) = (x - 2)(x - 4)$

What x's make $(x - 2) = 0$ or $(x - 4) = 0$

$x = 2$ or $x = 4$

Zeros/x-int: x = 2 and x = 4 or (2, 0) and (4, 0)

Vertex Form: $f(x) = (x - 3)^2 - 1$

Vertex: (+3, -1)

Compare the vertex to vertex form. Notice anything?

For the intervals below, write as LOWEST, HIGHEST.

The domain is x, and the range is y.

The x's are unlimited: -forever to +forever

Domain: $-\infty, \infty$ **Range:** $-1, \infty$

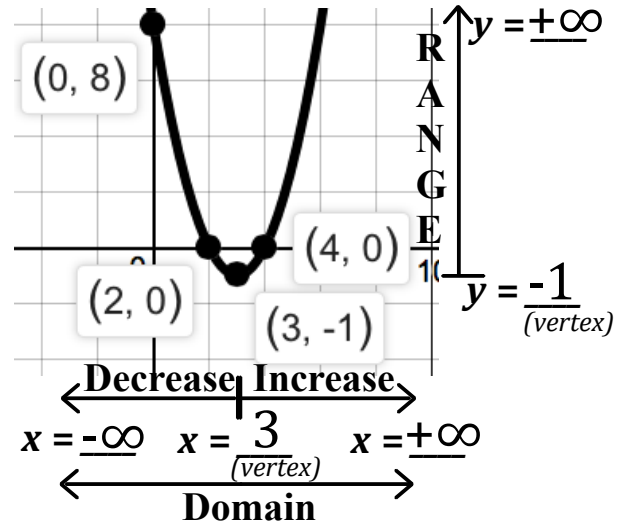
The y's are limited below by the vertex: $y = -1$ to +forever

The part that goes UP is on the right, starting at the vertex, where $x = 3$.

Interval of Increase: $3, \infty$

Interval of Decrease: $-\infty, 3$

The part that goes DOWN is on the left, ending at the vertex, where $x = 3$.



The only place where a quadratic stops is at the vertex (turning point). The rest of the time, it continues forever.

So, to describe its movement, I use the infinity symbol:

$-\infty$ means smallest forever & $+\infty$ means biggest forever

EXAMPLE 2. Standard form: $f(x) = -5x^2 + 10x + 15$

$f(x) = -5x^2 + 10x + 15$

y-int: (0, 15) or y = 15

Factored form: $f(x) = -5(x - 3)(x + 1)$

What x's make $(x - 3) = 0$ or $(x + 1) = 0$?

$x = 3$ or $x = -1$

Zeros/x-int: x = -1 and x = 3 or (-1, 0) and (3, 0)

Vertex Form: $f(x) = -5(x - 1)^2 + 20$

Vertex: (1, 20)

Compare the vertex to vertex form. Notice anything?

For the intervals below, write as LOWEST, HIGHEST.

The domain is x, and the range is y.

The x's are unlimited: -forever to +forever

Domain: $-\infty, \infty$ **Range:** $-\infty, 20$

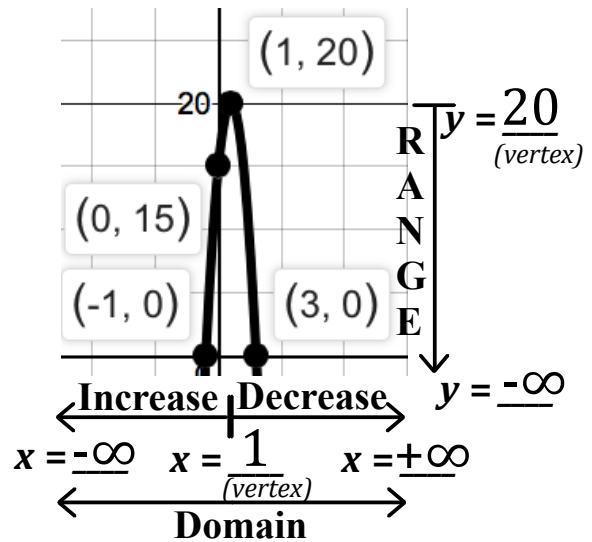
The y's are limited above by the vertex: -forever to $y = 20$

The part that goes UP is on the left, ending at the vertex, where $x = 1$.

Interval of Increase: $-\infty, 1$

Interval of Decrease: $1, \infty$

The part that goes DOWN is on the right, starting at the vertex, where $x = 1$.



1. Standard form: $f(x) = -x^2 + 6x - 8$

y-int: _____

Factored form: $f(x) = -(x - 2)(x - 4)$

Zeros/x-int: _____

Vertex Form: $f(x) = -(x - 3)^2 + 1$

Vertex: _____

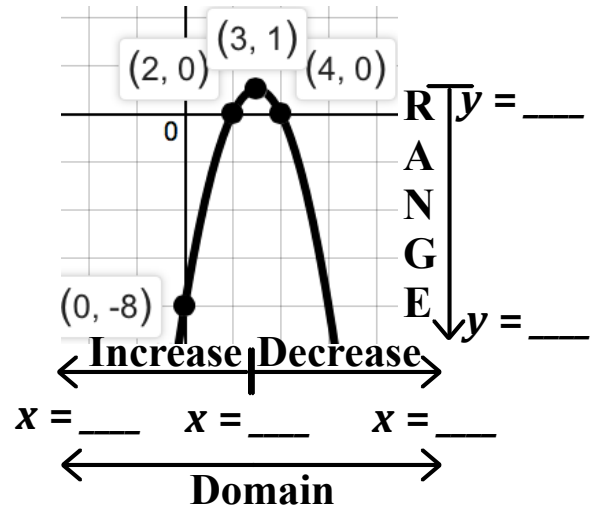
Compare the vertex to vertex form. Notice anything?

For the intervals below, write as LOWEST, HIGHEST.

Domain: _____, _____ Range: _____, _____

Interval of _____ Interval of _____

Increase: _____, _____ Decrease: _____, _____



2. Standard form: $f(x) = x^2 + 2x - 3$

y-int: _____

Factored form: $f(x) = (x + 3)(x - 1)$

Zeros/x-int: _____

Vertex Form: $f(x) = (x + 1)^2 - 4$

Vertex: _____

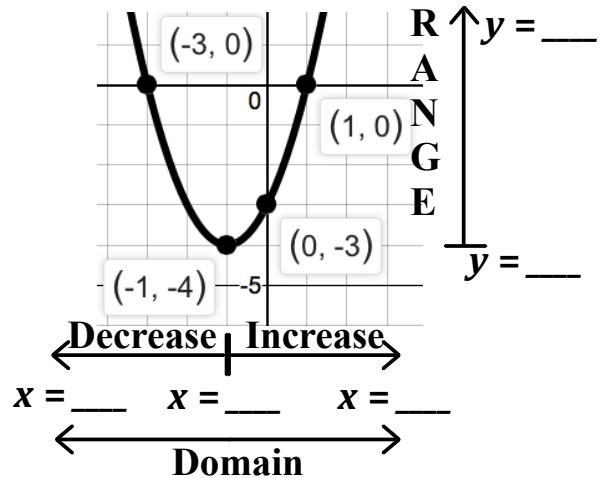
Compare the vertex to vertex form. Notice anything?

For the intervals below, write as LOWEST, HIGHEST.

Domain: _____, _____ Range: _____, _____

Interval of _____ Interval of _____

Increase: _____, _____ Decrease: _____, _____



3. Standard form: $f(x) = 4x^2 + 24x + 20$

y-int: _____

Factored form: $f(x) = 4(x + 1)(x + 5)$

Zeros/x-int: _____

Vertex Form: $f(x) = 4(x + 3)^2 - 16$

Vertex: _____

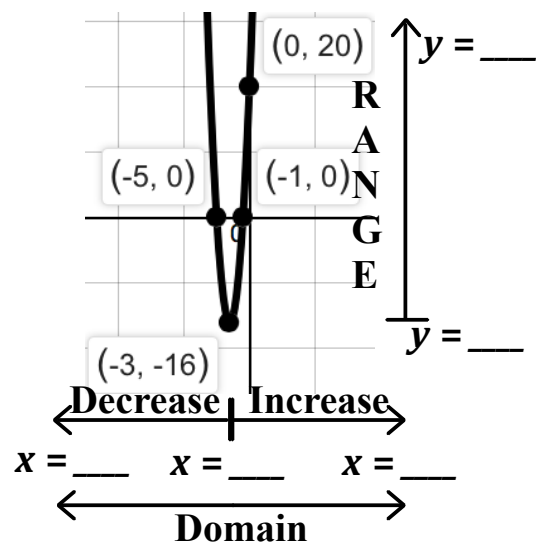
Compare the vertex to vertex form. Notice anything?

For the intervals below, write as LOWEST, HIGHEST.

Domain: _____, _____ Range: _____, _____

Interval of _____ Interval of _____

Increase: _____, _____ Decrease: _____, _____



4. Standard form: $f(x) = -x^2 - 8x - 7$

y-int: _____

Factored form: $f(x) = -(x + 7)(x + 1)$

Zeros/x-int: _____

Vertex Form: $f(x) = -(x + 4)^2 + 9$

Vertex: _____

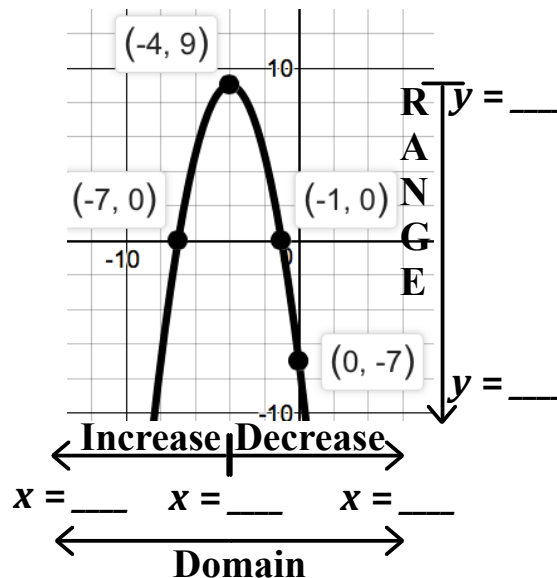
Compare the vertex to vertex form. Notice anything?

For the intervals below, write as LOWEST, HIGHEST.

Domain: _____, _____ Range: _____, _____

Interval of _____ Interval of _____

Increase: _____, _____ Decrease: _____, _____



5. Standard form: $f(x) = -3x^2 + 24x - 21$

y-int: _____

Factored form: $f(x) = -3(x - 1)(x - 7)$

Zeros/x-int: _____

Vertex Form: $f(x) = -3(x - 4)^2 + 27$

Vertex: _____

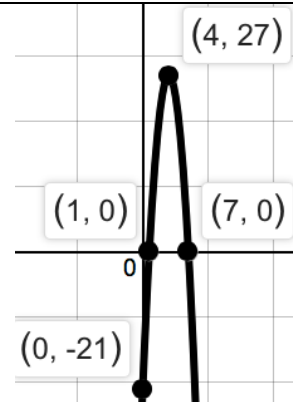
Compare the vertex to vertex form. Notice anything?

For the intervals below, write as LOWEST, HIGHEST.

Domain: _____, _____ Range: _____, _____

Interval of _____ Interval of _____

Increase: _____, _____ Decrease: _____, _____



6. Standard form: $f(x) = x^2 + 8x + 7$

y-int: _____

Factored form: $f(x) = (x + 7)(x + 1)$

Zeros/x-int: _____

Vertex Form: $f(x) = (x + 4)^2 - 9$

Vertex: _____

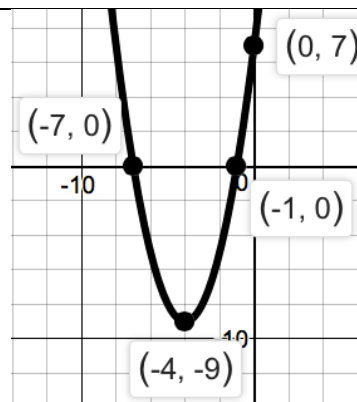
Compare the vertex to vertex form. Notice anything?

For the intervals below, write as LOWEST, HIGHEST.

Domain: _____, _____ Range: _____, _____

Interval of _____ Interval of _____

Increase: _____, _____ Decrease: _____, _____



Now, we need to work on using correct interval notation. So far, you have written your DOMAIN, RANGE, Interval of INCREASE and Interval of DECREASE as “lowest, highest.” While this format is correct, it is also incomplete.

The incomplete notation you have used tells us where the interval starts and where it ends. But it does not tell us whether the start and end are part of the interval (“included”) or not part of the interval.

Think of it like “greater than or equal to” instead of simply “greater than”

[&] mean “or equal to,” whereas (&) mean “but not equal to”

Translate our inequality symbols using the examples below.

EXAMPLE 3. $-2 \leq x \leq 5$ Interval: $[-2, 5]$	EXAMPLE 4. $-2 < x < 5$ Interval: $(-2, 5)$	EXAMPLE 5. $-2 < x \leq 5$ Interval: $(-2, 5]$	7. $-2 \leq x < 5$ Interval:	8. $3 < x \leq 7$ Interval:
9. $3 \leq x \leq 7$ Interval:	10. $3 \leq x < 7$ Interval:	11. $3 < x < 7$ Interval:	12. $-8 < y \leq -4$ Interval:	13. $-8 \leq y \leq -4$ Interval:

Nothing can be equal to infinity, which means that ∞ will never be included.

Meaning: Infinity will **always** be $(-\infty \text{ or } \infty)$ and **never** next to $[-\infty \text{ or } \infty]$

14. $-\infty < x \leq 5$ Interval:	15. $-\infty < x < 5$ Interval:	16. $7 < y < \infty$ Interval:	17. $7 < y < \infty$ Interval:	18. $-\infty < x < \infty$ Interval:
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For **intervals of increase and decrease**, the vertex (turning point) is neither increasing nor decreasing, so it will not be included in either.

Meaning: Intervals of increase and decrease are always $(-\infty, \text{vertex})$ or (vertex, ∞)

19. Increase: $6, \infty$ should be: $__ 6, \infty __$	20. Decrease: $-\infty, 6$ should be: $__ -\infty, 6 __$	21. Increase: $-\infty, -4$ should be: $__ -\infty, -4 __$	22. Decrease: $-4, \infty$ should be: $__ -4, \infty __$
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For **domain and range**, the vertex is a part of the graph, which means it is included in both the domain and the range.

Meaning: the domain and range will always be either: $(-\infty, \infty)$, $[\text{vertex}, \infty)$ or $(-\infty, \text{vertex}]$

23. Domain: $-\infty, \infty$ should be: $__ -\infty, \infty __$	24. Range: $5, \infty$ should be: $__ 5, \infty __$	25. Range: $-\infty, 7$ should be: $__ -\infty, 7 __$	26. Domain: $-\infty, \infty$ should be: $__ -\infty, \infty __$
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Using the notation that you now know, go back to questions 1-6 and put the correct brackets on the Domain, Range and Intervals of Increase and Decrease.