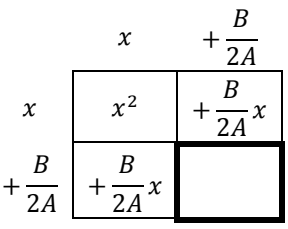
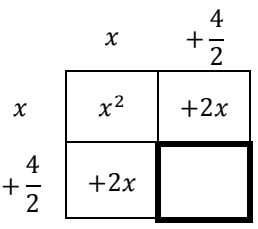
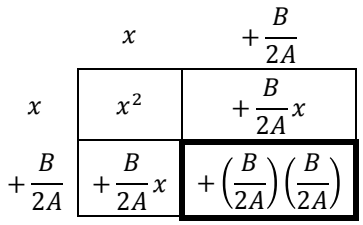
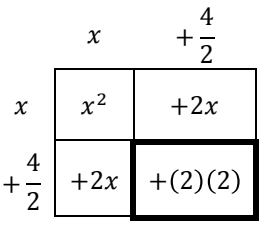
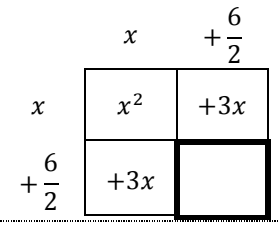
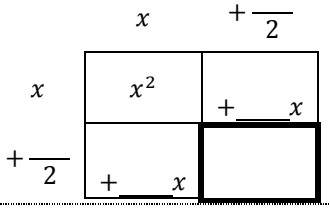
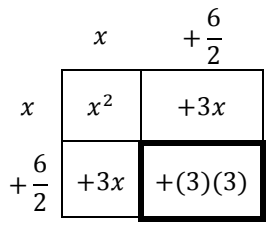
Name: _____

Completing the Square – Vertex Form

There are 3 traditional equation forms for a quadratic: Standard Form, Factored Form, and Vertex Form. We already know how to create Factored Form (factor the standard form). Today, we will be creating Vertex Form.

The process for creating Vertex Form from Standard Form is called "Completing the Square."

The Process	**Example 1**
Start with Standard Form: $f(x) = Ax^2 + Bx + C$	Start with Standard Form: $f(x) = 3x^2 + 12x + 8$
Step 1: Group $(Ax^2 + Bx)$ $f(x) = (Ax^2 + Bx) + C$	Step 1: Group $(Ax^2 + Bx)$ $f(x) = (3x^2 + 12x) + 8$
Step 2: Factor A out of $(Ax^2 + Bx)$ $f(x) = A\left(\frac{A}{A}x^2 + \frac{B}{A}x\right) + C$ *Simplify what you can:* $f(x) = A\left(x^2 + \frac{B}{A}x\right) + C$	Step 2: Factor A out of $(Ax^2 + Bx)$ $f(x) = 3\left(\frac{3}{3}x^2 + \frac{12}{3}x\right) + 8$ *Simplify what you can:* $f(x) = 3(x^2 + 4x) + 8$
Step 3: Rewrite the equation to **add** a *box* **inside** the group and **subtract** *A times a box* **outside** the group: $f(x) = A\left(x^2 + \frac{B}{A}x + \square\right) + C - (A)(\square)$	Step 3: Rewrite the equation to **add** a *box* **inside** the group and **subtract** *A times a box* **outside** the group: $f(x) = 3\left(x^2 + 4x + \square\right) + 8 - (3)(\square)$
Step 4: Create a multiplication square for the group  In order to create a SQUARE, you will have to divide the x-term's number by 2 The sides will be $x + \frac{B}{2A}$	Step 4: Create a multiplication square for the group  In order to create a SQUARE, you will have to divide the x-term's number by 2 The sides will be $x + \frac{4}{2}$
Step 5: Complete the square, and put that number in the two boxes from the equation.  $f(x) = A\left(x^2 + \frac{B}{A}x + \boxed{\left(\frac{B}{2A}\right)^2}\right) + C - (A)\left(\boxed{\left(\frac{B}{2A}\right)^2}\right)$	Step 5: Complete the square, and put that number in the two boxes from the equation.  $f(x) = 3\left(x^2 + 4x + \boxed{(2)^2}\right) + 8 - (3)\left(\boxed{(2)^2}\right)$ *Simplified:* $f(x) = 3\left(x^2 + 4x + \boxed{4}\right) + 8 - (3)\left(\boxed{4}\right)$
Step 6: Write the equation group as a square – use the outside of the multiplication square. $f(x) = A\left(x + \frac{B}{2A}\right)^2 + C - (A)\left(\left(\frac{B}{2A}\right)^2\right)$	Step 6: Write the equation group as a square – use the outside of the multiplication square. $f(x) = 3\left(x + \frac{4}{2}\right)^2 + 8 - (3)(4)$
Step 7: Simplify as much as possible. $f(x) = A\left(x + \frac{B}{2A}\right)^2 + C - (A)\left(\frac{B^2}{4A^2}\right)$ $f(x) = A\left(x + \frac{B}{2A}\right)^2 + C - \frac{B^2}{4A}$	Step 7: Simplify as much as possible. $f(x) = 3(x + 2)^2 + 8 - 12$ $\boxed{\text{Vertex form: } f(x) = 3(x + 2)^2 - 4}$

Standard form:	**Example 2**: $g(x) = 4x^2 + 24x - 7$	1. $h(x) = 2x^2 + 8x - 1$
Step 1:	Group $(Ax^2 + Bx)$ $g(x) = (4x^2 + 24x) - 7$	
Step 2:	Factor A out of $(Ax^2 + Bx)$ $g(x) = 4\left(\frac{4}{4}x^2 + \frac{24}{4}x\right) - 7$ *Simplify what you can:* $g(x) = 4(x^2 + 6x) - 7$	
Step 3:	Rewrite it to **add** a *box* **inside** the group and **subtract** *A times a box* **outside** the group: $g(x) = 4(x^2 + 6x + \square) - 7 - (4)(\square)$	
Step 4:	Create a multiplication square.  Divide the x-term's number by 2 The sides will be $x + \frac{6}{2}$	
Step 5:	Complete the square, and put that number in the two boxes from the equation.  $g(x) = 4\left(x^2 + 6x + \boxed{(3)^2}\right) - 7 - (4)\left(\boxed{(3)^2}\right)$ *Simplified:* $g(x) = 4(x^2 + 6x + \boxed{9}) - 7 - (4)(\boxed{9})$	
Step 6:	Write the equation group as a square – use the outside of the multiplication square. $g(x) = 4\left(x + \frac{6}{2}\right)^2 - 7 - (4)(9)$	
Step 7:	Simplify as much as possible. $g(x) = 4(x + 3)^2 - 7 - 36$ Vertex form: $g(x) = 4(x + 3)^2 - 43$	

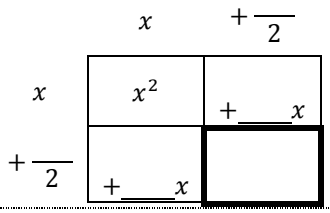
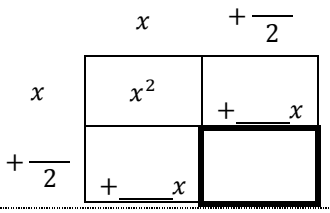
Which step(s) are the most confusing? Find a classmate that can explain the confusing steps to you.

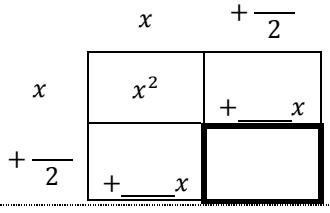
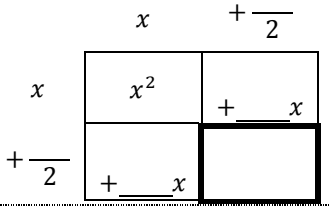
Which step(s) make the most sense to you? Find a classmate who does not understand these steps and help them to understand.

Once you have gotten and given the needed help, continue to practice completing the square.

Standard form:	**Example 3:** $k(x) = -2x^2 + 20x + 6$	2. $f(x) = -3x^2 + 6x + 4$
Step 1:	$k(x) = (-2x^2 + 20x) + 6$	
Step 2:	$k(x) = -2\left(\frac{-2}{-2}x^2 + \frac{20}{-2}x\right) + 6$ $k(x) = -2(x^2 - 10x) + 6$	
Step 3:	$k(x) = -2\left(x^2 - 10x + \square\right) + 6 - (-2)(\square)$	
Step 4:	x | $+\frac{-10}{2}$ x | x^2 | $-5x$ $+\frac{-10}{2}$ | $-5x$ | ← in step 5, this will be $(-5)(-5)$	x | $+\frac{\quad}{2}$ x | x^2 | $+\_\_\_x$ $+\frac{\quad}{2}$ | $+\_\_\_x$ |
Step 5:	$k(x) = -2\left(x^2 - 10x + \boxed{(-5)^2}\right) + 6 - (-2)\left(\boxed{(-5)^2}\right)$ $k(x) = -2\left(x^2 - 10x + \boxed{25}\right) + 6 - (-2)\left(\boxed{25}\right)$	
Step 6:	$k(x) = -2\left(x + \frac{-10}{2}\right)^2 + 6 - (-2)(25)$	
Step 7:	$k(x) = -2(x + -5)^2 + 6 - (-50)$ $k(x) = -2(x - 5)^2 + 6 + 50$ Vertex form: $k(x) = -2(x - 5)^2 + 56$	

Standard form:	3. $g(x) = 5x^2 - 30x - 4$	4. $h(x) = -x^2 - 8x + 3$ *Hint:* $A = -1$
Step 1:		
Step 2:		
Step 3:		
Step 4:	x | $+\frac{\quad}{2}$ x | x^2 | $+\_\_\_x$ $+\frac{\quad}{2}$ | $+\_\_\_x$ |	x | $+\frac{\quad}{2}$ x | x^2 | $+\_\_\_x$ $+\frac{\quad}{2}$ | $+\_\_\_x$ |
Step 5:		
Step 6:		
Step 7:		

Standard form:	5. $k(x) = x^2 + 12x - 9$ *Hint:* $A = 1$	6. $m(x) = 4x^2 - 8x + 7$
Step 1:		
Step 2:		
Step 3:		
Step 4:		
Step 5:		
Step 6:		
Step 7:		

Standard form:	7. $n(x) = -4x^2 - 8x + 7$	8. $h(x) = x^2 - 20x + 4$
Step 1:		
Step 2:		
Step 3:		
Step 4:		
Step 5:		
Step 6:		
Step 7:		