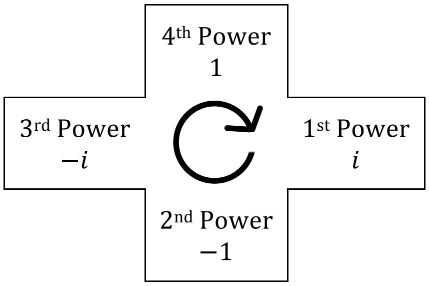
Name: ______________________________

The Circle of *i*



Today, we are going to be working with imaginary numbers, represented by the letter, *i*. Imaginary numbers follow a pattern, based on their exponents.

To the left is what I refer to as the **Circle of *i***.
The first four powers of *i* each have a different value:

$i^1 = i \quad i^2 = -1 \quad i^3 = -i \quad \& \; i^4 = 1$

but, after the 4th power, the pattern repeats itself:

$i^5 = i \quad i^6 = -1 \quad i^7 = -i \quad \& \; i^8 = 1$

$i^9 = i \quad i^{10} = -1 \quad i^{11} = -i \quad \& \; i^{12} = 1$

Try using the circular pattern to evaluate the powers of *i*.

1. $i^1 = ?$	2. $i^2 = ?$	3. $i^3 = ?$	4. $i^4 = ?$
5. $i^5 = ?$	6. $i^6 = ?$	7. $i^7 = ?$	8. $i^8 = ?$
9. $i^9 = ?$	10. $i^{10} = ?$	11. $i^{11} = ?$	12. $i^{12} = ?$
13. $i^{13} = ?$	14. $i^{14} = ?$	15. $i^{15} = ?$	16. $i^{16} = ?$

What this means is that, if you know the pattern, then you can simplify *i* to any power. There are three ways to do this:

1. Count around the circle until you find the power that you need (which is the easiest way, if the exponent is small)
   (For example, if I want to simplify i^{247}, then I will start at the 1st power and count around the circle until I get to the 247th power, which lands on the 3rd power's place. So, $i^{247} = i^3 = \boxed{-i}$)

or

2. Use the 4th power as an anchor and only start counting at a multiple of 4 that is close to the given exponent.
   (For example, if I want to simplify i^{247}, I know that 240 is divisible by 4 and close to 247, so I will start counting there: 240 is the 4th power, …*BACK TO 1st*… 241 is 1st, 242 is 2nd, 243 is 3rd, 244 is 4th, …*BACK TO 1st*…245 is 1st, 246 is 2nd and 247 is 3rd. Now I know that i^{247} has the same value as i^3.
   So, $i^{247} = i^3 = \boxed{-i}$)

or

3. Divide the exponent by 4, but STOP dividing at the decimal point. Whatever the REMAINDER is, that is your exponent (if there is no remainder, then the exponent belongs on the 4th power).
   (For example, if I want to simplify i^{247}, then I will divide 247 by 4, stopping at the decimal. I know that the number that is left at the bottom when I hit the decimal and cannot bring down anything more is called my REMAINDER. That number replaces my exponent of 247.

```
    061.
 4 | 247. STOP AT THE DECIMAL!
    -0
    -24
    -24
     07
    - 4
      3   This is your REMAINDER!
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$i^{247} = i^{REMAINDER}$

$i^{247} = i^3 = \boxed{-i}$

Choose whichever method you like best, and simplify each imaginary number below.

17. $i^{25} = ?$	18. $i^{39} = ?$	19. $i^{34} = ?$	20. $i^{20} = ?$
21. $i^{100} = ?$	22. $i^{56} = ?$	23. $i^{87} = ?$	24. $i^{105} = ?$
25. $i^{147} = ?$	26. $i^{123} = ?$	27. $i^2 = ?$	28. $i^9 = ?$

Up until today, you had only worked with "Real Numbers," which are every number in the world that it is possible to create. There is, however, a set of "Imaginary Numbers," which are the numbers created when you square root a negative number – an impossible task, mathematically, at least if the numbers are real. Whenever we find a square root with a negative inside, we know that the number is not a real number, so we must rewrite it as an imaginary one. To do this, we remove the negative from inside the root and put the letter *i* out in front of it. Practice below.

EXAMPLE: $\sqrt{-3} = \boxed{i\sqrt{3}}$	29. $\sqrt{-7}$	30. $\sqrt{-10}$	31. $\sqrt{-5}$
EXAMPLE: $\sqrt{-14} = \boxed{i\sqrt{14}}$	32. $\sqrt{-6}$	33. $\sqrt{-15}$	34. $\sqrt{-22}$

If you can simplify the square root that's left, then simplify it, and put the number in front of *i*.

EXAMPLE: $\sqrt{-4} = i\sqrt{4} = i(2)$ $= \boxed{2i}$	35. $\sqrt{-25}$	36. $\sqrt{-9}$	37. $\sqrt{-100}$
EXAMPLE: $\sqrt{-144} = i\sqrt{144} = i(12)$ $= \boxed{12i}$	38. $\sqrt{-49}$	39. $\sqrt{-81}$	40. $\sqrt{-225}$