Name: ______________________________

Fractional Exponents

A fractional exponent is simply a root expressed as an exponent. Let's start with rewriting roots as exponents.

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|---|---|---|
| **Example** $\sqrt{x} = x^{\frac{1}{2}}$ | 1. $\sqrt{5} = 5^{?}$ | 2. $\sqrt{a} = a^{?}$ |
| **Example** $\sqrt[3]{x} = x^{\frac{1}{3}}$ | 3. $\sqrt[5]{x} = x^{?}$ | 4. $\sqrt[4]{8} = 8^{?}$ |

This is why a root cancels an exponent:

$$\sqrt{x^2} = (x^2)^{\frac{1}{2}} = x^{(2)\left(\frac{1}{2}\right)} = x^1 = x$$

Generally, we do not go through all of that work. We simply accept that they cancel each other and follow the rule.

The next thing that we need to understand is what a fractional exponent is asking us to do. We all know that $x^3$ means that we want to multiply by $x$ three times, but what do we want if it says $x^{\frac{1}{3}}$? It's not division…so what do we do? There is no simple straight-forward process that gets your answer every time – it's a thought puzzle. $x^{\frac{1}{3}}$ is asking you to find what would multiply by itself three times to find $x$.

| | | |
|---|---|---|
| **Example** $256^{\frac{1}{4}} = ?$ | $256 = \underline{\ ?\ } \cdot \underline{\ ?\ } \cdot \underline{\ ?\ } \cdot \underline{\ ?\ }$<br>*What times itself 4 times will equal 256?*<br>$1 \cdot 1 \cdot 1 \cdot 1 = (1)(1) = 1,$<br>$2 \cdot 2 \cdot 2 \cdot 2 = (4)(4) = 16,$<br>$3 \cdot 3 \cdot 3 \cdot 3 = (9)(9) = 81,$<br>$4 \cdot 4 \cdot 4 \cdot 4 = (16)(16) = 256$ ***found it!*** | $256^{\frac{1}{4}} = \boxed{4}$<br>*because* $4 \cdot 4 \cdot 4 \cdot 4 = 256$ |
| 5. $27^{\frac{1}{3}} = ?$ | $27 = \underline{\ ?\ } \cdot \underline{\ ?\ } \cdot \underline{\ ?\ }$<br>*What times itself 3 times will equal 27?*<br>*Stop when you find it!*<br>$1 \cdot 1 \cdot 1 =$ $\quad$ $2 \cdot 2 \cdot 2 =$<br>$3 \cdot 3 \cdot 3 =$ $\quad$ $4 \cdot 4 \cdot 4 =$<br>$5 \cdot 5 \cdot 5 =$ $\quad$ … | $27^{\frac{1}{3}} =$ |
| 6. $32^{\frac{1}{5}} = ?$ | $32 = \underline{\ ?\ } \cdot \underline{\ ?\ } \cdot \underline{\ ?\ } \cdot \underline{\ ?\ } \cdot \underline{\ ?\ }$<br>*What times itself 5 times will equal 32?*<br>*Stop when you find it!*<br>$1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 =$ $\quad$ $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 =$<br>$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 =$ $\quad$ $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 =$<br>$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 =$ $\quad$ … | $32^{\frac{1}{5}} =$ |

| | | |
|---|---|---|
| 7. $64^{\frac{1}{3}}$ | 8. $100{,}000^{\frac{1}{5}}$ | 9. $125^{\frac{1}{3}}$ |
| 10. $81^{\frac{1}{4}}$ | 11. $729^{\frac{1}{3}}$ | 12. $64^{\frac{1}{6}}$ |
| 13. $8^{\frac{1}{3}}$ | 14. $243^{\frac{1}{5}}$ | 15. $64^{\frac{1}{2}}$ |

Now that we know how to simplify our fractional exponents, lets look at how to determine the value of *x* when we know what raising it to a power will equal. **For the purposes of this practice, we will assume that all roots are positive.**

**Example**

Rewrite the problem using a fractional exponent, then determine the value of *x*, assuming that *x* is a positive number.

$$x^4 = 625$$

*is the same as*

$$x \cdot x \cdot x \cdot x = 625,$$

*which means that*

$$x = 625^{\frac{1}{4}}$$

*What times itself 4 times will get me 625?*

$$1 \cdot 1 \cdot 1 \cdot 1 = 1 \quad nope$$
$$2 \cdot 2 \cdot 2 \cdot 2 = 16 \quad nope$$
$$\ldots$$
$$5 \cdot 5 \cdot 5 \cdot 5 = 625!!$$

So, if $x^4 = 625$, then $x = 625^{\frac{1}{4}} = \boxed{5}$

**Example**

Rewrite the problem using a fractional exponent, then determine the value of *x*, assuming that *x* is a positive number.

$$x^3 = 512$$

$$x \cdot x \cdot x = 512,$$

$$x = 512^{\frac{1}{3}}$$

$$1 \cdot 1 \cdot 1 = 1 \quad nope$$
$$2 \cdot 2 \cdot 2 = 8 \quad nope$$
$$\ldots$$
$$8 \cdot 8 \cdot 8 = 512!!$$

So, if $x^3 = 512$, then $x = 512^{\frac{1}{3}} = \boxed{8}$

16. Rewrite the problem using a fractional exponent, then determine the value of *x*, assuming that *x* is a positive number.

$$x^4 = 16$$

17. Rewrite the problem using a fractional exponent, then determine the value of *x*, assuming that *x* is a positive number.

$$x^8 = 1$$

18. Rewrite the problem using a fractional exponent, then determine the value of *x*, assuming that *x* is a positive number.

$$x^3 = 216$$

19. Rewrite the problem using a fractional exponent, then determine the value of *x*, assuming that *x* is a positive number.

$$x^2 = 49$$

20. Rewrite the problem using a fractional exponent, then determine the value of *x*, assuming that *x* is a positive number.

$$x^4 = 256$$

21. Rewrite the problem using a fractional exponent, then determine the value of *x*, assuming that *x* is a positive number.

$$x^3 = 1000$$

22. Rewrite the problem using a fractional exponent, then determine the value of *x*, assuming that *x* is a positive number.

$$x^4 = 81$$

23. Rewrite the problem using a fractional exponent, then determine the value of *x*, assuming that *x* is a positive number.

$$x^3 = 125$$

**Answers**

| | | | | |
|---|---|---|---|---|
| 1. $\frac{1}{2}$ | 2. $\frac{1}{2}$ | 3. $\frac{1}{5}$ | 4. $\frac{1}{4}$ | 5. $27^{\frac{1}{3}} = \boxed{3}$ |
| 6. $32^{\frac{1}{5}} = \boxed{2}$ | 7. $64^{\frac{1}{3}} = \boxed{4}$ | 8. $100{,}000^{\frac{1}{5}} = \boxed{10}$ | 9. $125^{\frac{1}{3}} = \boxed{5}$ | 10. $81^{\frac{1}{4}} = \boxed{3}$ |
| 11. $729^{\frac{1}{3}} = \boxed{9}$ | 12. $64^{\frac{1}{6}} = \boxed{2}$ | 13. $8^{\frac{1}{3}} = \boxed{2}$ | 14. $243^{\frac{1}{5}} = \boxed{3}$ | 15. $64^{\frac{1}{2}} = \boxed{8}$ |

| | | | |
|---|---|---|---|
| 16. $16^{\frac{1}{4}} = 2$ | 17. $1^{\frac{1}{8}} = 1$ | 18. $216^{\frac{1}{3}} = 6$ | 19. $49^{\frac{1}{2}} = 7$ |
| 20. $256^{\frac{1}{4}} = 4$ | 21. $1000^{\frac{1}{3}} = 10$ | 22. $81^{\frac{1}{4}} = 3$ | 23. $125^{\frac{1}{3}} = 5$ |