

Polynomial Operations

Adding & Subtracting

When there is a symbol (+ or -) between the parentheses, that means that you are not multiplying the polynomials together – you’re adding or subtracting them.

There are three addition/subtraction rules that you need to remember:

1. The sign in front of the parentheses belongs to every part – you have to distribute the symbol.  
*(think of it as multiplying each part by +1 or -1, if that helps)*
2. You will add and subtract only the front numbers (called *coefficients*). When you are adding/subtracting the variable and its exponents *do not change*.
3. You can only combine like terms – terms that have the same variable with the same exponent.  
*(think of  $3x^3 + 5x^3$  like you’re saying, “I have 3 x-cubes and another 5 x-cubes, which means I have 8 x-cubes. So,  $3x^3 + 5x^3 = 8x^3$ .)*

**Add or subtract the polynomials.**

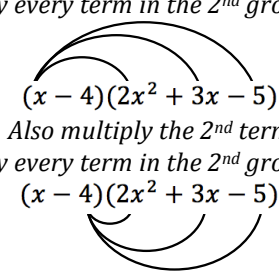
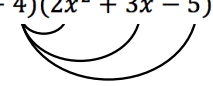
	<b>EXAMPLE</b> $-(6x^3 + x - 3) + (4x^2 - 3x - 12)$	1. $-(8x^3 + 2x^2 + 10x) + (-2x^3 + 6x - 1)$							
Distribute the sign in front of each ( )	$-(6x^3) - (+x) - (-3) + (4x^2) + (-3x) + (-12)$ $-6x^3 - x + 3 + 4x^2 - 3x - 12$								
Combine like terms	<i>Organize your information (keep their signs)</i>	How many $x^3$ s?    How many $x^2$ s?    How many $x$ 's?    How many numbers?							
	<table border="1"> <tr> <td>How many <math>x^3</math>s?</td> <td>How many <math>x^2</math>s?</td> <td>How many <math>x</math>'s?</td> <td>How many numbers?</td> </tr> <tr> <td><math>-6x^3</math></td> <td><math>+4x^2</math></td> <td><math>-x</math> &amp; <math>-3x</math></td> <td><math>+3</math> &amp; <math>-12</math></td> </tr> </table> <p><i>Combine your terms</i> <math>-6x^3 + 4x^2 - x - 3x + 3 - 12</math> <math>-6x^3 + 4x^2 - 4x - 9</math></p>	How many $x^3$ s?	How many $x^2$ s?	How many $x$ 's?	How many numbers?	$-6x^3$	$+4x^2$	$-x$ & $-3x$	$+3$ & $-12$
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$-6x^3$	$+4x^2$	$-x$ & $-3x$	$+3$ & $-12$						

	<b>EXAMPLE</b> $(-4x^4 - 9x^2 + 7x) - (5x^2 - 7x + 5)$	2. $(-2x^4 - 7x^3 - 12) - (x^3 + 10x - 11)$																				
Distribute the sign in front of each ( )	$+(-4x^4) + (-9x^2) + (+7x) - (5x^2) - (-7x) - (+5)$ $-4x^4 - 9x^2 + 7x - 5x^2 + 7x - 5$																					
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3. $-(3x^2 + 2x - 1) - (4x^3 - 3x + 5)$	4. $-(5x^4 + 2x + 4) + (2x^3 + 3x)$	5. $(-2x^2 - 4x + 11) - (5x^4 + 8x^2 + 9)$
6. $(6x^2 - x - 1) + (4x^3 - 9x^2 - 5x)$	7. $(10x^3 + 7x) - (4x^3 - 3x^2 - x + 8)$	8. $(9x^3 + 12x - 9) - (7x^2 - 6x + 9)$

### Multiplying

There are 3 traditional methods for multiplying polynomials:

<p style="text-align: center;"><u>Distribution</u></p> $(x - 4)(2x^2 + 3x - 5)$ <p style="text-align: center;"><i>Split the 2<sup>nd</sup> group &amp; copy the 1<sup>st</sup>:</i></p> $2x^2(x - 4) + 3x(x - 4) - 5(x - 4)$ <p style="text-align: center;"><i>Distribute:</i></p> $2x^3 - 8x^2 + 3x^2 - 12x - 5x + 20$ <p style="text-align: center;"><i>Combine like terms:</i></p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: auto;"><math>2x^3 - 5x^2 - 17x + 20</math></div>	<p style="text-align: center;"><u>Box Method</u></p> $(x - 4)(2x^2 + 3x - 5)$ <p style="text-align: center;"><i>Create a box with the 1<sup>st</sup> group on the side and the 2<sup>nd</sup> on the top</i></p> <table style="margin: auto;"> <tr> <td></td> <td style="text-align: center;"><math>2x^2</math></td> <td style="text-align: center;"><math>+3x</math></td> <td style="text-align: center;"><math>-5</math></td> </tr> <tr> <td style="text-align: center;"><math>x</math></td> <td style="border: 1px solid black; width: 40px; height: 20px;"></td> <td style="border: 1px solid black; width: 40px; height: 20px;"></td> <td style="border: 1px solid black; width: 40px; height: 20px;"></td> </tr> <tr> <td style="text-align: center;"><math>-4</math></td> <td style="border: 1px solid black; width: 40px; height: 20px;"></td> <td style="border: 1px solid black; width: 40px; height: 20px;"></td> <td style="border: 1px solid black; width: 40px; height: 20px;"></td> </tr> </table> <p style="text-align: center;"><i>Use it like a multiplication table:</i></p> <table style="margin: auto;"> <tr> <td></td> <td style="text-align: center;"><math>2x^2</math></td> <td style="text-align: center;"><math>+3x</math></td> <td style="text-align: center;"><math>-5</math></td> </tr> <tr> <td style="text-align: center;"><math>x</math></td> <td style="border: 1px solid black; width: 40px; height: 20px;"><math>2x^3</math></td> <td style="border: 1px solid black; width: 40px; height: 20px;"><math>+3x^2</math></td> <td style="border: 1px solid black; width: 40px; height: 20px;"><math>-5x</math></td> </tr> <tr> <td style="text-align: center;"><math>-4</math></td> <td style="border: 1px solid black; width: 40px; height: 20px;"><math>-8x^2</math></td> <td style="border: 1px solid black; width: 40px; height: 20px;"><math>-12x</math></td> <td style="border: 1px solid black; width: 40px; height: 20px;"><math>+20</math></td> </tr> </table> <p style="text-align: center;"><i>Now, combine the like terms in the middle.</i></p> $2x^3 - 8x^2 + 3x^2 - 12x - 5x + 20$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: auto;"><math>2x^3 - 5x^2 - 17x + 20</math></div>		$2x^2$	$+3x$	$-5$	$x$				$-4$					$2x^2$	$+3x$	$-5$	$x$	$2x^3$	$+3x^2$	$-5x$	$-4$	$-8x^2$	$-12x$	$+20$	<p style="text-align: center;"><u>FOIL (or Rainbow)</u></p> $(x - 4)(2x^2 + 3x - 5)$ <p style="text-align: center;"><i>Multiply the 1<sup>st</sup> term of the 1<sup>st</sup> group by every term in the 2<sup>nd</sup> group.</i></p>  $(x - 4)(2x^2 + 3x - 5)$ <p style="text-align: center;"><i>Also multiply the 2<sup>nd</sup> term by every term in the 2<sup>nd</sup> group.</i></p> $(x - 4)(2x^2 + 3x - 5)$  $x(2x^2) + x(+3x) + x(-5) - 4(2x^2) - 4(+3x) - 4(-5)$ <p style="text-align: center;"><i>Simplify.</i></p> $2x^3 + 3x^2 - 5x - 8x^2 - 12x + 20$ <p style="text-align: center;"><i>Combine like terms:</i></p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: auto;"><math>2x^3 - 5x^2 - 17x + 20</math></div>
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$x$	$2x^3$	$+3x^2$	$-5x$																							
$-4$	$-8x^2$	$-12x$	$+20$																							
9. $(3x - 4)(x^2 + 5x - 4)$	10. $(5x - 7)(x + 3)$	11. $(x + 9)(x^2 - 9x + 81)$																								
12. $(5x + 3)(5x - 3)$	13. $(x - 3)(-4x^2 - 2x + 6)$	14. $(x + 4)(2x^2 - 6x + 2)$																								

### Dividing Polynomials by Monomial Factors

When a polynomial is the numerator (top) of a fraction and there is a monomial in the denominator (bottom) that can be divided from all of its terms, you can simplify the fraction by dividing each term by the bottom (distribute the division). If only part is divisible, then divide each term by that, and leave what isn't divisible in the denominator.

<p><b>EXAMPLE</b></p> $\frac{-3x^3 + 6x^2 + 24x}{3x}$ <p style="text-align: center;"><i>All terms can be divided by 3x.</i></p> $\frac{-3x^3 \div 3x \quad + 6x^2 \div 3x \quad + 24x \div 3x}{-1x^2 + 2x + 8}$	<p><b>EXAMPLE</b></p> $\frac{7x^4 + 4x^3 - 5x^2}{2x^2}$ <p style="text-align: center;"><i>All terms can be divided by x<sup>2</sup>, but not by 2.</i></p> $\frac{7x^4 \div x^2 \quad + 4x^3 \div x^2 \quad - 5x^2 \div x^2}{\frac{7x^2 + 4x - 5}{2}}$	<p><b>EXAMPLE</b></p> $\frac{16x^2 + 8x - 12}{4x}$ <p style="text-align: center;"><i>All terms can be divided by 4, but not by x.</i></p> $\frac{16x^2 \div 4 \quad + 8x \div 4 \quad - 12 \div 4}{\frac{4x^2 + 2x - 3}{x}}$
15. $\frac{9x^4 + 81x^3 - 18x^2}{3x^2}$	16. $\frac{-6x^5 + 24x^4 - 16x^3}{5x^3}$	17. $\frac{8x^3 + 2x^2 - 6x}{2x^2} \leftarrow \text{think of as } 2 \cdot x \cdot x$