Name: ______________________________

Solving Systems of Linear Equations: Elimination

Solve the system using elimination twice. First, solve by eliminating *x*, and then solve it again using *y*.

	Eliminate x	*Eliminate for y*
System	1a. $\begin{cases} -2y = 5x + 11 \\ y + 3x = -8 \end{cases}$	1b. $\begin{cases} -2y = 5x + 11 \\ y + 3x = -8 \end{cases}$
Pick an equation to rewrite so that the *x*'s, the *y*'s, the numbers and the ='s are all lined up (it doesn't matter which equation).		
Pick either *x* or *y* to eliminate, then multiply one (or both) of the equations by whatever numbers will make the elimination variables equal but opposite (one +, one -) to each other.		
Add down to create a new equation. Then, solve it.		
Plug the variable you found into either one of the original equations. Then, solve for the other variable.		
Write the answer as a point (x, y).		

Solve each system using substitution (you decide if you want to start by solving for *x* or for *y*).

2. $\begin{cases} 9y + 43 = -x \\ -8y = 7x + 26 \end{cases}$	3. $\begin{cases} -2x + 5y = 0 \\ 4x - 6y = 16 \end{cases}$	4. $\begin{cases} 8x = -4y - 32 \\ -2x - y = 8 \end{cases}$
5. $\begin{cases} 2y - 38 = -4x \\ -y - 1 = x \end{cases}$	6. $\begin{cases} -4y = -x + 2 \\ 3x - 12y = -2 \end{cases}$	7. $\begin{cases} -7x + 11 = -9y \\ 5x + 5 = 5y \end{cases}$

Answers

1. $(-5, 7)$	2. $(2, -5)$	3. $(10, 4)$	4. $Inf. Many$	5. $(6, -7)$	6. $No\ Sol.$	7. $(-10, -9)$

Solving Systems of Linear Equations: Substitution

Solve the system using substitution twice. First, solve by substituting for *x*, and then solve it again using *y*.

	Eliminate x	*Eliminate for y*
System	8a. $\begin{cases} -2y = 5x + 11 \\ y + 3x = -8 \end{cases}$	8b. $\begin{cases} -2y = 5x + 11 \\ y + 3x = -8 \end{cases}$
Pick an equation and isolate either *x* or *y* (SADMEP).		
Plug what the isolated variable equals into the <u>other</u> original equation. Then solve it.		
Plug the variable you found into either one of the original equations. Then, solve for the other variable.		
Write the answer as a point (x, y).		

Solve each system using substitution (you decide if you want to start by solving for *x* or for *y*).

9. $\begin{cases} 9y + 43 = -x \\ -8y = 7x + 26 \end{cases}$	10. $\begin{cases} -2x + 5y = 0 \\ 4x - 6y = 16 \end{cases}$	11. $\begin{cases} 8x = -4y - 32 \\ -2x - y = 8 \end{cases}$
12. $\begin{cases} 2y - 38 = -4x \\ -y - 1 = x \end{cases}$	13. $\begin{cases} -4y = -x + 2 \\ 3x - 12y = -2 \end{cases}$	14. $\begin{cases} -7x + 11 = -9y \\ 5x + 5 = 5y \end{cases}$

Answers

8. $(-5, 7)$	9. $(2, -5)$	10. $(10, 4)$	11. $Inf. Many$	12. $(6, -7)$	13. $No\ Sol.$	14. $(-10, -9)$

Solving Systems of Linear Equations

Elimination Example

A system of equations is a set of equations that might cross somewhere. The solutions to the system, if there are any, are any points (or point) where the two graphs cross. There are two options to solve systems algebraically: elimination and substitution.

Elimination is the process of lining up two equations above each other, then setting up the equations so that, when you add them down, one of the variables cancels out, allowing you to solve to for the remaining variable. Once you have it, you go back to original equations and plug that variable's value into one of them in order to determine the other variable. Then, you just write the answer as a point.

EXAMPLE:

<table>
<tr><th></th><th>Eliminate x</th><th>Eliminate y</th></tr>
<tr><td rowspan="2">Pick an equation to rewrite so that the x's, the y's, the numbers and the ='s are all lined up (it doesn't matter which equation).</td><td>$\begin{cases} y = 5x + 18 \\ -6y - 8x = -32 \end{cases}$</td><td>$\begin{cases} y = 5x + 18 \\ -6y - 8x = -32 \end{cases}$</td></tr>
<tr><td>I can choose either one, so I'm going to change the top equation. It needs to match the order of $-6y - 8x = 32$, so I will move the x term to the left side of the =.
$y = 5x + 18$
$\underline{-5x \quad -5x}$
$y - 5x = 18$
System now looks like:
$\begin{cases} y - 5x = 18 \\ -6y - 8x = -32 \end{cases}$</td><td>I can choose either one, so I'm going to change the top equation. It needs to match the order of $-6y - 8x = 32$, so I will move the x term to the left side of the =.
$y = 5x + 18$
$\underline{-5x \quad -5x}$
$y - 5x = 18$
System now looks like:
$\begin{cases} y - 5x = 18 \\ -6y - 8x = -32 \end{cases}$</td></tr>
<tr><td>Pick either x or y to eliminate, then multiply one (or both) of the equations by whatever numbers will make the elimination variables equal but opposite (one +, one -) to each other.</td><td>If I want to eliminate x, I need to make
$-5x$ & $-8x$
into matching opposites (to cancel).
$-5x \cdot 8 = -40x,$
$\& -8x \cdot -5 = -40x$

So, I will multiply the entire top equation by 8 & the bottom equation by -5.

$8(y - 5x = 18) \rightarrow \quad 8y - 40x = 144$
$-5(-6y - 8x = -32) \rightarrow 30y + 40x = 160$</td><td>If I want to eliminate y, I need to make
y & $-6y$
into matching opposites (to cancel).
$y \cdot 6 = 6y,$
$\& -6y \cdot 1 = -6y$

So, I will multiply the entire top equation by 6 & the bottom equation by 1.

$6(y - 5x = 18) \rightarrow \quad 6y - 30x = 108$
$1(-6y - 8x = -32) \rightarrow -6y - 8x = -32$</td></tr>
<tr><td>Add down to create a new equation. Then, solve it.</td><td>$\begin{array}{rrcr} & 8y & -40x & = & 144 \\ + & 30y & +40x & = & 160 \\ \hline & 38y & +0x & = & 304 \end{array}$
$38y = 304$
$\underline{\div 38 \quad \div 38}$
$y = 8$</td><td>$\begin{array}{rrcr} & 6y & -30x & = & 108 \\ + & -6y & -8x & = & -32 \\ \hline & 0y & -38x & = & 76 \end{array}$
$-38x = 76$
$\underline{\div -38 \quad \div -38}$
$x = -2$</td></tr>
<tr><td colspan="3">If the new equation is something that's always true, like $\boldsymbol{x = x, or\ y = y, or -5 = -5,}$ then STOP!
The answer is $\boxed{\boldsymbol{Infinitely\ many\ solutions}}$
If, on the other hand, the equation is something that's impossible, like $\boldsymbol{-3 = 7, or\ 1 = 2, or\ 0 = 10}$, then STOP!
The answer is $\boxed{\boldsymbol{No\ solution}}$</td></tr>
<tr><td>Plug the variable you found into either one of the original equations. Then, solve for the other variable.</td><td>$y = 5x + 18$ The first equation looks easier.
$(plug\ in\ here) = 5x + 18$
$8 = 5x + 18$
$\underline{-18 \qquad -18}$
$-10 = 5x$
$\underline{\div 5 \qquad \div 5}$
$-2 = x$
$x = -2$</td><td>$y = 5x + 18$ The first equation looks easier.
$y = 5(plug\ in\ here) + 18$
$y = 5(-2) + 18$
$y = -10 + 18$
$y = 8$</td></tr>
<tr><td>Write the answer as a point (x, y).</td><td>$x = -2$ & $y = 8$, so the answer is:
$\boxed{(-2, 8)}$</td><td>$x = -2$ & $y = 8$, so the answer is:
$\boxed{(-2, 8)}$</td></tr>
</table>

Solving Systems of Linear Equations

Substitution Example

Substitution is the process of solving one of the equations for either *x* or *y* and then plug what it equals (the entire expression) in for that variable in the <u>other</u> equation, allowing you to solve to for the remaining variable. Once you have it, you go back to original equations and plug that variable's value into one of them in order to determine the other variable. Then, you just write the answer as a point.

EXAMPLE:

	Substitute for x	*Substitute for y*
	$\begin{cases} y=5x+18 \\ -6y-8x=-32 \end{cases}$	$\begin{cases} y=5x+18 \\ -6y-8x=-32 \end{cases}$
Pick an equation and isolate either *x* or *y* (SADMEP).	$y=5x+18$ *The first equation looked easier* *(I could have chosen either one, though).* *Isolate x:* $y=5x+18$ $\underline{-18 \quad -18}$ $y-18=5x$ $\underline{\div 5 \quad \div 5 \quad \div 5}$ $\frac{y}{5}-\frac{18}{5}=x$ $x=\frac{y}{5}-\frac{18}{5}$	$y=5x+18$ *The first equation is already solved for y, so it is the best choice. If I used the second one, I would have to divide everything by -6 (which would work, but makes the problem more difficult)* *Isolate y:* $y=5x+18$ *...it's already isolated, so that's done.*
Plug what the isolated variable equals into the <u>other</u> original equation. Then solve it.	$-6y-8x=-32 \leftarrow the\ other\ equation$ $-6y-8(plug\ in\ here)=-32$ $-6y-8\left(\frac{y}{5}-\frac{18}{5}\right)=-32$ *Distribute* $-6y-\frac{8y}{5}+\frac{144}{5}=-32$ *Trick: get rid of all of the fractions by multiplying every term by the denominator: 5!* $-6y\cdot 5-\frac{8y}{\cancel{5}}\cdot\frac{\cancel{5}}{1}+\frac{144}{\cancel{5}}\cdot\frac{\cancel{5}}{1}=-32\cdot 5$ *Simplify* $-30y-8y+144=-160$ *Solve for y.* $-30y-8y+144=-160$ $-38y+144=-160$ $\underline{\quad -144 \quad -144}$ $-38y=-304$ $\underline{\div -38 \quad \div -38}$ $y=8$	$-6y-8x=-32 \leftarrow the\ other\ equation$ $-6(plug\ in\ here)-8x=-32$ $-6(5x+18)-8x=-32$ *Distribute* $-30x-108-8x=-32$ *Simplify* $-38x-108=-32$ *Solve for x.* $-38x-108=-32$ $\underline{\quad +108 \ +108}$ $-38x=76$ $\underline{\div -38 \quad \div -38}$ $x=-2$
If the new equation is something that's always true, like $\boldsymbol{x=x, or\ y=y, or\ -5=-5,}$ then **STOP!** The answer is $\boxed{\boldsymbol{Infinitely\ many\ solutions}}$ If, on the other hand, the equation is something that's impossible, like $\boldsymbol{-3=7, or\ 1=2, or\ 0=10}$, then **STOP!** The answer is $\boxed{\boldsymbol{No\ solution}}$		
Plug the variable you found into either one of the original equations. Then, solve for the other variable.	$y=5x+18$ *The first equation looks easier.* $(plug\ in\ here)=5x+18$ $8=5x+18$ $\underline{-18 \quad -18}$ $-10=5x$ $\underline{\div 5 \quad \div 5}$ $-2=x$ $x=-2$	$y=5x+18$ *The first equation looks easier.* $y=5(plug\ in\ here)+18$ $y=5(-2)+18$ $y=-10+18$ $y=8$
Write the answer as a point (x,y).	$x=-2$ & $y=8$, so the answer is: $\boxed{(-2,8)}$	$x=-2$ & $y=8$, so the answer is: $\boxed{(-2,8)}$