Name: \_\_\_\_\_ Solving Systems of Linear Equations: Elimination

## Solve the system using elimination twice. First, solve by eliminating *x*, and then solve it again using *y*.

	Eliminate x	Eliminate for y
System	1a. $\begin{cases} -2y = 5x + 11\\ y + 3x = -8 \end{cases}$	1b. $\begin{cases} -2y = 5x + 11\\ y + 3x = -8 \end{cases}$
Pick an equation to rewrite so that the x's, the y's, the numbers and the ='s are all lined up (it doesn't matter which equation).		
Pick either x or y to eliminate, then multiply one (or both) of the equations by whatever numbers will make the elimination variables equal but opposite (one +, one -) to each other.		
Add down to create a new equation. Then, solve it.		
Plug the variable you found into either one of the original equations. Then, solve for the other variable.		
Write the answer as a point ( <i>x</i> , <i>y</i> ).		

## Solve each system using substitution (you decide if you want to start by solving for x or for y).

(9y + 43 = -r)	(-2r + 5y - 0)	(8r = -4v - 32)		
2. $\begin{cases} 9y + 43 = -x \\ -8y = 7x + 26 \end{cases}$	3. $\begin{cases} -2x + 5y = 0\\ 4x - 6y = 16 \end{cases}$	4. $\begin{cases} 8x = -4y - 32 \\ -2x - y = 8 \end{cases}$		
(-8y = 7x + 26)	(4x - 6y = 16)	(-2x - y = 8)		
(2y - 38 = -4x)	(-4y = -x + 2)	(-7x + 11 = -9y)		
5. $\begin{cases} 2y - 38 = -4x \\ -y - 1 = x \end{cases}$	6. $\begin{cases} -4y = -x + 2\\ 3x - 12y = -2 \end{cases}$	7. $\begin{cases} -7x + 11 = -9y \\ 5x + 5 = 5y \end{cases}$		
(-y-1) = x	(3x - 12y = -2)	(5x + 5 = 5y)		
Answers				
1. (-5,7) 2. (2,-5) 3. (1		6. <i>No Sol</i> . 7. (-10, -9)		
		( -, -, -,		

# Solving Systems of Linear Equations: Substitution

	<i>Eliminate x</i>	Eliminate for y
System	8a. $\begin{cases} -2y = 5x + 11\\ y + 3x = -8 \end{cases}$	8b. $\begin{cases} -2y = 5x + 11\\ y + 3x = -8 \end{cases}$
Pick an equation and isolate either <i>x</i> or <i>y</i> (SADMEP).		
Plug what the isolated variable equals into the <u>other</u> original equation. Then solve it.		
Plug the variable you found into either one of the original equations. Then, solve for the other variable.		
Write the answer as a point ( <i>x</i> , <i>y</i> ).		

# Solve the system using substitution twice. First, solve by substituting for *x*, and then solve it again using *y*.

### Solve each system using substitution (you decide if you want to start by solving for x or for y).

Solve each system using substitution	h (you decide if you want to start by so	
9. $\begin{cases} 9y + 43 = -x \\ -8y = 7x + 26 \end{cases}$	10. $\begin{cases} -2x + 5y = 0\\ 4x - 6y = 16 \end{cases}$	11. $\begin{cases} 8x = -4y - 32 \\ -2x - y = 8 \end{cases}$
9. $(-8y = 7x + 26)$	10. $(4x - 6y = 16)$	11. $(-2x - y = 8)$
12. $\begin{cases} 2y - 38 = -4x \\ -y - 1 = x \end{cases}$	(-4y = -x + 2)	(-7x + 11 = -9y)
$12. \left\{ -y - 1 = x \right\}$	13. $\begin{cases} -4y = -x + 2\\ 3x - 12y = -2 \end{cases}$	14. $\begin{cases} -7x + 11 = -9y \\ 5x + 5 = 5y \end{cases}$

Answers						
8. (-5,7)	9. (2, -5)	10. (10, 4)	11. Inf. Many	12. (6, -7)	13. No Sol.	14. (-10, -9)

#### Solving Systems of Linear Equations Elimination Example

A system of equations is a set of equations that might cross somewhere. The solutions to the system, if there are any, are any points (or point) where the two graphs cross. There are two options to solve systems algebraically: elimination and substitution.

Elimination is the process of lining up two equations above each other, then setting up the equations so that, when you add them down, one of the variables cancels out, allowing you to solve to for the remaining variable. Once you have it, you go back to original equations and plug that variable's value into one of them in order to determine the other variable. Then, you just write the answer as a point.

Eliminate xEliminate yPick an equation to rewrite so that the x's, the y's, the numbers and the ='s are all lined up (it doesn't matter which equation).I can choose either one, so I'm going to change the top equation. It needs to match the order of $-6y - 8x = 32$ , so I will move the x term to the left side of the =. $y = 5x + 18$ $-5x = 5x$ $y - 5x = 18$ System now looks like: $\begin{cases} y - 5x = 18\\ -6y - 8x = -32 \end{cases}$ I can choose either one, so I'm going to the top equation. It needs to match the order of $-6y - 8x = 32$ , so I will move the x term to the left side of the =. $y = 5x + 18$ $-5x = -5x$ $y - 5x = 18$ System now looks like: $\begin{cases} y - 5x = 18\\ -6y - 8x = -32 \end{cases}$ I can choose either one, so I'm going to the top equation. It needs to match the $-6y - 8x = -32$ Pick either x or y to eliminate, then multiply one (orIf I want to eliminate x, I need to make $-5x \& -8x$ If I want to eliminate y, I need to make $y \& -6y$			
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Pick either x or y to eliminate, thenIf I want to eliminate x, I need to make $-5x \& -8x$ If I want to eliminate y, I need to make $y \& -6y$			
eliminate, then $-5x \& -8x$ $y \& -6y$			
	e		
multiply one for intermetating ennesites (to cancel) into matching ennesites (to can			
	ncelJ.		
both) of the $-5x \cdot 8 = -40x$ , $y \cdot 6 = 6y$ ,			
equations by $\& -8x \cdot -5 = -40x \qquad \& -6y \cdot 1 = -6y$			
whatever numbers			
will make the So, I will multiply the entire top equation by 8 So, I will multiply the entire top equat			
elimination & the bottom equation by -5. the bottom equation by 1.	the bottom equation by 1.		
variables equal but			
opposite (one +, $8(y - 5x = 18) \rightarrow 8y - 40x = 144$ $6(y - 5x = 18) \rightarrow 6y - 30x = 18$	$6(y - 5x = 18) \rightarrow 6y - 30x = 108$		
one -) to each $-5(-6y - 8x = -32) \rightarrow 30y + 40x = 160$ $1(-6y - 8x = -32) \rightarrow -6y - 8x = -32$	$1(-6y - 8x = -32) \rightarrow -6y - 8x = -32$		
other.			
$8y - 40x = 144 \qquad 6y - 30x = 108$			
$\frac{+30y + 40x = 160}{+ - 6y - 8x = -32}$			
Add down to create $38y + 0x = 304$ $0y - 38x = 76$			
a new equation.			
Then, solve it. $38y = 304$ $-38x = 76$			
$\frac{\div 38 \div 38}{\div -38} \qquad \qquad \frac{\div -38 \div -38}{\div -38}$			
$y = 8 \qquad \qquad x = -2$			
If the new equation is something that's always true, like $x = x$ , or $y = y$ , or $-5 = -5$ , then <b>STOP</b>	P!		
The answer is <i>Infinitely many solutions</i>			
If, on the other hand, the equation is something that's impossible, like $-3 = 7$ , or $1 = 2$ , or $0 = 10$ , then	n STOP!		
The answer is <b>No solution</b>			
y = 5x + 18 The first equation looks easier. $y = 5x + 18$ The first equation look	ks easier		
Plug the variable $(plug in here) = 5x + 18$ $y = 5(plug in here) + 18$	no cusici.		
	y = 5(-2) + 18		
	y = -10 + 18		
	y = 8		
Then, solve for the $\frac{\div 5 \div 5}{2 - \pi}$			
other variable. $-2 = x$			
x = -2			
Write the answer $x = -2$ & $y = 8$ , so the answer is: $x = -2$ & $y = 8$ , so the answer is:			
as a point $(x, y)$ . $(-2, 8)$			

EXAMPLE:

#### Solving Systems of Linear Equations Substitution Example

Substitution is the process of solving one of the equations for either *x* or *y* and then plug what it equals (the entire expression) in for that variable in the <u>other</u> equation, allowing you to solve to for the remaining variable. Once you have it, you go back to original equations and plug that variable's value into one of them in order to determine the other variable. Then, you just write the answer as a point.

	EXAMPLE:					
	Substitute for x	Substitute for y				
	y = 5x + 18	y = 5x + 18				
	(-6y - 8x = -32)	$\int -6y - 8x = -32$				
	y = 5x + 18	y = 5x + 18				
	The first equation looked easier	The first equation is already solved for y, so it is				
	(I could have chosen either one, though).	the best choice. If I used the second one, I would				
Dislama a succession	Isolate x:	have to divide everything by -6 (which would				
Pick an equation	y = 5x + 18	work, but makes the problem more difficult)				
and isolate either <i>x</i>	-18 - 18					
or y (SADMEP).	y - 18 = 5x	Isolate y:				
	$\frac{\frac{\div 5}{y} \div 5 \div 5}{\frac{y}{5} - \frac{18}{5}} = x$ $x = \frac{y}{5} - \frac{18}{5}$	y = 5x + 18				
	$\overline{y}$ 18	it's already isolated, so that's done.				
	$\frac{1}{5} - \frac{1}{5} = x$					
	y 18					
	$x = \frac{5}{5} - \frac{1}{5}$					
	$-6y - 8x = -32 \leftarrow the other equation$	$-6y - 8x = -32 \leftarrow the other equation$				
	-6y - 8(plug in here) = -32	-6(plug in here) - 8x = -32				
		-6(5x + 18) - 8x = -32				
	$-6y - 8\left(\frac{y}{5} - \frac{18}{5}\right) = -32$	Distribute				
	Distribute	-30x - 108 - 8x = -32				
	8y 144	Simplify				
	$-6y - \frac{8y}{5} + \frac{144}{5} = -32$	-38x - 108 = -32				
Plug what the	Trick: get rid of all of the fractions by	Solve for x.				
isolated variable	multiplying every term by the denominator: 5!	-38x - 108 = -32				
equals into the	8v = 5 144 =	-100 = 32 +108 + 108				
other original	$-6y \cdot 5 - \frac{8y}{5} \cdot \frac{5}{1} + \frac{144}{5} \cdot \frac{5}{1} = -32 \cdot 5$	$\frac{-100 + 100}{-38x = 76}$				
equation.	Simplify					
Then solve it.	-30y - 8y + 144 = -160	$\frac{\div -38 \div -38}{x = -2}$				
Then solve it.	Solve for y.	x = -2				
	-30y - 8y + 144 = -160					
	-38y + 144 = -160					
	-144 - 144					
	$\frac{-38y = -304}{-38y = -304}$					
	$\div -38 \div -38$					
	$\frac{1}{y=8}$					
If the new (	equation is something that's always true, like $x =$	- x  or  y - y  or  - 55  then STOP				
II the new t						
The answer is <i>Infinitely many solutions</i>						
It, on the other ha	nd, the equation is something that's impossible,					
	The answer is <b>No solut</b>					
	y = 5x + 18 The first equation looks easier.	y = 5x + 18 The first equation looks easier.				
Plug the variable	(plug in here) = 5x + 18	y = 5(plug in here) + 18				
you found into	8 = 5x + 18	y = 5(-2) + 18				
either one of the	-18 - 18	y = -10 + 18				
original equations.	-10 = 5x	y = 8				
Then, solve for the	<u>÷5 ÷5</u>					
other variable.	$\overline{-2 = x}$					
	x = -2					
Write the answer	x = -2 & $y = 8$ , so the answer is:	x = -2 & $y = 8$ , so the answer is:				
as a point $(x, y)$ .	[(-2,8)]	[(-2,8)]				
us u point ( <i>x</i> , <i>y</i> ).						

#### EXAMPLE: