Name: _____

Factoring with Perfect Squares to Simplify Radicals

Every number has at least 1 set of factors: 1 times the original number. Most numbers are not prime, which means that they have more than one set of factors. Today, we will be searching for perfect square factors, meaning factors that can be square rooted. We are trying to find factors that are on the this list:

	$1^2 =$	$2^2 =$	$3^2 =$	$4^2 =$	$5^2 =$	$6^2 =$	$7^2 =$	$8^2 =$	$9^2 =$	$10^2 =$
Perfect Squares:	**1**	**4**	**9**	**16**	**25**	**36**	**49**	**64**	**81**	**100**

	$11^2 =$	$12^2 =$	$13^2 =$	$14^2 =$	$15^2 =$	$16^2 =$	$17^2 =$	$18^2 =$	$19^2 =$	$20^2 =$
Perfect Squares:	**121**	**144**	**169**	**196**	**225**	**256**	**289**	**324**	**361**	**400**

List all of the factors of each number and determine if any of them are perfect.

EXAMPLE: Factor 54.

Perfect?	Factor	Factor	Perfect?
Yes	1	54	No
No	2	27	No
No	3	18	No
	~~4~~		
	~~5~~		
No	6	9	Yes
	~~7~~		
	~~8~~		

The list has met in the middle – the next number is 9, which is on the bottom of the 2nd list- so I can stop.

What are the factor sets with perfect squares?

Yes: $54 = (1)(54)$ *or* $(6)(9)$

EXAMPLE: Factor 120.

Perfect?	Factor	Factor	Perfect?
Yes	1	120	No
No	2	60	No
No	3	40	No
Yes	4	30	No
No	5	24	No
No	6	20	No
	~~7~~		
No	8	15	No
	~~9~~		
No	10	12	No
	~~11~~		

What are the factor sets with perfect squares?

Yes: $120 = (1)(120)$ *or* $(4)(30)$

1. Factor 50.

Perfect?	Factor	Factor	Perfect?
	1		
	2		
	3		
	4		
	5		

Factor sets with perfect squares:

2. Factor 76.

Perfect?	Factor	Factor	Perfect?
	1		
	2		
	3		
	4		
	5		
	6		
	7		
	8		
	9		

Factor sets with perfect squares:

3. Factor 75.

Perfect?	Factor	Factor	Perfect?
	1		
	2		
	3		
	4		
	5		

Since I know that there are no more factors of 75, I can stop here. If I was not sure, I would have to stop until the list meets in the middle.

Factor sets with perfect squares:

4. Factor 40.

Perfect?	Factor	Factor	Perfect?

Factor sets with perfect squares:

5. Factor 15.

Perfect?	Factor	Factor	Perfect?

Factor sets with perfect squares:

6. Factor 24.

Perfect?	Factor	Factor	Perfect?

Factor sets with perfect squares:

7. Factor 20.

Perfect?	Factor	Factor	Perfect?

Factor sets with perfect squares:

8. Factor 28.

Perfect?	Factor	Factor	Perfect?

Factor sets with perfect squares:

9. Factor 18.

Perfect?	Factor	Factor	Perfect?

Factor sets with perfect squares:

10. Factor 99.

Factor sets with perfect squares:

11. Factor 44.

Factor sets with perfect squares:

12. Factor 8.

Factor sets with perfect squares:

The purpose of this exercise has been to learn to identify perfect square factors in order to split square root radicals into two factors: one that can be simplified and one that cannot. **Square roots can be factored, just like regular numbers can. The only difference is that the square root factors will also be square roots.**

EXAMPLE: $\sqrt{12}$

	Factor	Factor	
Yes	$\sqrt{1}$	$\sqrt{12}$	No
No	$\sqrt{2}$	$\sqrt{6}$	No
No	$\sqrt{3}$	$\sqrt{4}$	Yes

Of these three factor sets, two have parts that can square root:

$$\sqrt{12} = \sqrt{1}\sqrt{12} = 1\sqrt{12}$$

but, that changes nothing…

$$\sqrt{12} = \sqrt{3}\sqrt{4} = \sqrt{3}(2) = 2\sqrt{3}$$

So, when simplified, $\sqrt{12}$ is $\boxed{2\sqrt{3}}$.

EXAMPLE: $\sqrt{54}$

	Factor	Factor	
~~Yes~~	~~$\sqrt{1}$~~	~~$\sqrt{54}$~~	~~No~~
No	$\sqrt{2}$	$\sqrt{27}$	No
No	$\sqrt{3}$	$\sqrt{18}$	No
No	$\sqrt{6}$	$\sqrt{9}$	Yes

I know that square rooting 1 won't change anything, so I'm going to use the bigger square root:

$$\sqrt{54} = \sqrt{6}\sqrt{9} = \sqrt{6}(3) = \boxed{3\sqrt{6}}$$

EXAMPLE: $\sqrt{120}$

	Factor	Factor	
~~Yes~~	~~$\sqrt{1}$~~	~~$\sqrt{120}$~~	~~No~~
No	$\sqrt{2}$	$\sqrt{60}$	No
No	$\sqrt{3}$	$\sqrt{40}$	No
Yes	$\sqrt{4}$	$\sqrt{30}$	No
No	$\sqrt{5}$	$\sqrt{24}$	No
No	$\sqrt{6}$	$\sqrt{20}$	No
No	$\sqrt{8}$	$\sqrt{15}$	No
No	$\sqrt{10}$	$\sqrt{12}$	No

$$\sqrt{120} = \sqrt{4}\sqrt{30} = (2)\sqrt{30} = \boxed{2\sqrt{30}}$$

13. $\sqrt{50}$

See #1

14. $\sqrt{76}$

See #2

15. $\sqrt{125}$

See #3

16. $\sqrt{40}$ *See #4*	17. $\sqrt{15}$ *See #5*	18. $\sqrt{24}$ *See #6*
19. $\sqrt{20}$ *See #7*	20. $\sqrt{28}$ *See #8*	21. $\sqrt{18}$ *See #9*

If the square root has a negative inside, it's an imaginary number, which means we have to take out the negative by putting i in front of the radical. Then, follow the same process we've been using to simplify the square root.

Your answer should be written as: _? _$i\sqrt{}$_?__

22. $\sqrt{-99}$ $\sqrt{-99} = i\sqrt{99}$ $= i\sqrt{\quad}\sqrt{\quad}$ $= i(\quad)\sqrt{\quad}$ $= \quad i\sqrt{\quad}$ *See #10*	23. $\sqrt{-44}$ *See #11*	24. $\sqrt{-8}$ *See #12*

Check your Answers

1. $(1)(50)$ *or* $(2)(25)$	2. $(1)(76)$	3. $(1)(75)$ *or* $(3)(25)$
4. $(1)(40)$ *or* $(4)(10)$	5. $(1)(15)$	6. $(1)(24)$ *or* $(4)(6)$
7. $(1)(20)$ *or* $(4)(5)$	8. $(1)(28)$ *or* $(4)(7)$	9. $(1)(18)$ *or* $(2)(9)$
10. $(1)(99)$ *or* $(9)(11)$	11. $(1)(44)$ *or* $(4)(11)$	12. $(1)(8)$ *or* $(2)(4)$

13. $5\sqrt{2}$	14. $\sqrt{76}$	15. $5\sqrt{3}$
16. $2\sqrt{10}$	17. $\sqrt{15}$	18. $2\sqrt{6}$
19. $2\sqrt{5}$	20. $2\sqrt{7}$	21. $3\sqrt{2}$

22. $3i\sqrt{11}$	23. $2i\sqrt{11}$	24. $2i\sqrt{2}$