Determine Quadratic Solutions from Vertex and Factored Form

The solutions to a quadratic equation are also called the zeros or the x-intercepts, because the solutions are the values of *x* that make the equation (the *y*-value) equal to 0. This means that solving a quadratic is about setting it equal to 0.

**VERTEX FORM:**

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| **EXAMPLE:** $f\left(x\right)=5(x-3)^{2}+125$$$0=5(x-3)^{2}+125$$$$0=5(x-3)^{2}+125$$$$-125=5(x-3)^{2}$$$$-125=5(x-3)^{2}$$$$-25=(x-3)^{2}$$$$\pm \sqrt{-25}=\pm \sqrt{(x-3)^{2}}$$$$\pm i\sqrt{25}=(x-3)$$$$\pm 5i=x-3$$$$\pm 5i=x-3$$$$3\pm 5i=x$$**The zeros are:**$$x=3\pm 5i$$or: $\left(3-5i,0\right) and (3+5i, 0)$ | **EXAMPLE:** $f\left(x\right)=-3\left(x+2\right)^{2}+135$$$0=-3\left(x+2\right)^{2}+135$$$$0=-3\left(x+2\right)^{2}+135$$$$-135=-3\left(x+2\right)^{2}$$$$-135=-3\left(x+2\right)^{2}$$$$45=\left(x+2\right)^{2}$$$$\pm \sqrt{45}=\pm \sqrt{\left(x+2\right)^{2}}$$$$\pm \sqrt{5}\sqrt{9}=\left(x+2\right)$$$$\pm 3\sqrt{5}=x+2$$$$\pm 3\sqrt{5}=x+2$$$$-2\pm 3\sqrt{5}=x$$**The zeros are:**$$x=-2\pm 3\sqrt{5}$$or: $\left(-2-3\sqrt{5},0\right) and (-2+3\sqrt{5}, 0)$ | **EXAMPLE:** $f(x)=-(x-7)^{2}+44$$$0=-(x-7)^{2}+44$$$$0=-(x-7)^{2}+44$$$$-44=-(x-7)^{2}$$$$-44=-1(x-7)^{2}$$$$44=(x-7)^{2}$$$$\pm \sqrt{44}=\pm \sqrt{(x-7)^{2}}$$$$\pm \sqrt{4}\sqrt{11}=(x-7)$$$$\pm 2\sqrt{11}=x-7$$$$\pm 2\sqrt{11}=x-7$$$$7\pm 2\sqrt{11}=x$$**The zeros are:**$$x=7\pm 2\sqrt{11}$$or: $\left(7-2\sqrt{11},0\right) and (7+2\sqrt{11}, 0)$ |
| 1. $f\left(x\right)=-2\left(x+3\right)^{2}-18$$$0=-2\left(x+3\right)^{2}-18$$ | 2. $f\left(x\right)=3\left(x+5\right)^{2}+21$$$0=3\left(x+5\right)^{2}+21$$ | 3. $f\left(x\right)=-4\left(x-7\right)^{2}+100$$$0=-4\left(x-7\right)^{2}+100$$ |
| 4. $f\left(x\right)=5\left(x+23\right)^{2}-45$$$=5\left(x+23\right)^{2}-45$$ | 5. $f\left(x\right)=-\left(x-5\right)^{2}+14$$$=-\left(x-5\right)^{2}+14$$ | 6. $f\left(x\right)=8\left(x-47\right)^{2}-160$$$=8\left(x-47\right)^{2}-160$$ |

*Answers:* $1. \left(-3-3i, 0\right) \& (-3+3i, 0), 2. \left(-5-i\sqrt{7}, 0\right) \& (-5-i\sqrt{7}, 0), 3. \left(7-5, 0\right) \& \left(7+5, 0\right)=\left(2,0\right) \& (12, 0) $

$$4. \left(-23-3, 0\right) \& \left(-23+3, 0\right)=\left(-26, 0\right)\& (-20, 0), 5. \left(5-\sqrt{14}, 0\right)\& (5+\sqrt{14}, 0), 6. \left(47-2\sqrt{5}, 0\right)\& (47+2\sqrt{5})$$

**FACTORED FORM:**

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| **EXAMPLE:** $f(x)=(7x+2)(x+5)$$$0=(7x+2)(x+5)$$*This is actually two problems:*$$0=7x+2 and x+5=0$$*Add/subtract from both sides*$$-2=7x and x=-5$$*Multiply/divide from both sides*$$-\frac{2}{7}=x$$**The zeros are:**$$\left(-5, 0\right) and \left(-\frac{2}{7}, 0\right)$$ | **EXAMPLE:** $f(x)=2(x-6)(3x-8)$$$0=2(x-6)(3x-8)$$*This is actually two problems:* ***(ignore the number in front)***$$0=x-6 and 3x-8=0$$*Add/subtract from both sides*$$6=x and 3x=8$$*Multiply/divide from both sides*$$ x=\frac{8}{3}$$**The zeros are:**$$\left(\frac{8}{3}, 0\right) and \left(6, 0\right)$$ | **EXAMPLE:** $f(x)=-3(x+4)(x-7)$$$0=-3(x+4)(x-7)$$*This is actually two problems:* ***(ignore the number in front)***$$0=x+4 and x-7=0$$*Add/subtract from both sides*$$-4=x and x=7$$*~~Multiply/divide from both sides~~***The zeros are:**$$\left(-4, 0\right) and \left(7, 0\right)$$ |
| 7. $f(x)=(x-9)(3x+4)$$$0=(x-9)(3x+4)$$ | 8. $f\left(x\right)=-4(x+6)(2x-11)$$$0=-4(x+6)(2x-11)$$ | 9. $f\left(x\right)=-(x+9)(6x-5)$$$0=-(x+9)(6x-5)$$ |
| 10. $f\left(x\right)=-2(5x+1)(x-3)$$$=-2(5x+1)(x-3)$$ | 11. $f(x)=7(2x+3)(3x-4)$$$=7(2x+3)(3x-4)$$ | 12. $f\left(x\right)=-8(x+5)(x-4)$$$=-8(x+5)(x-4)$$ |

*Answers:* $7. \left(-\frac{4}{3},0\right) \& (9, 0), 8. \left(-6, 0\right) \& \left(\frac{11}{2},0\right), 9. \left(-9, 0\right) \& \left(\frac{5}{6}, 0\right) $

$10. \left(-\frac{1}{5},0\right) \& (3, 0), 11. \left(-\frac{3}{2},0\right) \& \left(\frac{4}{3},0\right), 12. \left(-5, 0\right) \& (4, 0)$

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*Now, we are going to practice plugging in values and simplifying square roots, using our DISCRIMINANT:*

$$\pm \sqrt{b^{2}-4ac}$$

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| **EXAMPLE:**$ Evaluate \pm \sqrt{b^{2}-4ac}$.$$a=5, b=-6, c=2$$$$\pm \sqrt{b^{2}-4ac}=\pm \sqrt{(-6)^{2}-4(5)(2)}$$$$=\pm \sqrt{36-4(10)}$$$$=\pm \sqrt{36-40}$$$$=\pm \sqrt{-4}$$$$=\pm 2i$$ | **EXAMPLE:**$ Evaluate \pm \sqrt{b^{2}-4ac}$.$$a=1, b=4, c=-9$$$$\pm \sqrt{b^{2}-4ac}=\pm \sqrt{(4)^{2}-4(1)(-9)}$$$$=\pm \sqrt{16-4(-9)}$$$$=\pm \sqrt{16+36}$$$$=\pm \sqrt{52}$$$$=\pm \sqrt{4}\sqrt{13}$$$$=\pm 2\sqrt{13}$$ | **EXAMPLE:**$ Evaluate \pm \sqrt{b^{2}-4ac}$.$$a=-3, b=12, c=-12$$$$\pm \sqrt{b^{2}-4ac}$$$$=\pm \sqrt{(12)^{2}-4(-3)(-12)}$$$$=\pm \sqrt{144-4(36)}$$$$=\pm \sqrt{144-144}$$$$=\pm \sqrt{0}$$$$=\pm 0$$$$=0$$ |
| 13. $Evaluate \pm \sqrt{b^{2}-4ac}$.$$a=1, b=3, c=-18$$ | 14. $Evaluate \pm \sqrt{b^{2}-4ac}$.$$a=2, b=-4, c=2$$ | 15. $Evaluate \pm \sqrt{b^{2}-4ac}$.$$a=10, b=9, c=3$$ |
| 16. $Evaluate \pm \sqrt{b^{2}-4ac}$.$$a=1, b=-7, c=12$$ | 17. $Evaluate \pm \sqrt{b^{2}-4ac}$.$$a=-3, b=9, c=-5$$ | 18. $Evaluate \pm \sqrt{b^{2}-4ac}$.$$a=1, b=6, c=8$$ |

*Answers:* $13. \pm 9, 14. 0, 15. \pm i\sqrt{39}, 16. \pm 1, 17. \pm \sqrt{21} 18. \pm 2$