Name: _____

Simplifying Imperfect Roots

Any number that has an integer square root is called a "perfect square." To square root a perfect square, you simplify have to remember what the root equals, and then write your answer (without the root symbol):

$\sqrt{1}=1$	$\sqrt{4}=2$	$\sqrt{9}=3$	$\sqrt{16}=4$	$\sqrt{25}=5$	$\sqrt{36}=6$	$\sqrt{49}=7$	$\sqrt{64}=8$	$\sqrt{81}=9$	$\sqrt{100}=10$
$\sqrt{121}=11$		$\sqrt{144}=12$		$\sqrt{169}=13$		$\sqrt{196}=14$		$\sqrt{225}=15$	
$\sqrt{256}=16$		$\sqrt{289}=17$		$\sqrt{324}=18$		$\sqrt{361}=19$		$\sqrt{400}=20$	

Square-rooting an *imperfect* number and getting an exact answer is not possible – at least not entirely. Your only option is to square-root the greatest perfect factor (see the list above) , and leave what can't be simplified inside a root.

EXAMPLE Simplify. $\sqrt{48}$

*You need the greatest **perfect** factor.*

48
$1\cdot 48$
$2\cdot 24$
$3\cdot \boxed{16}$ ←GPF
$4\cdot 12$
$6\cdot 8$

Then, rewrite the root as:
$\sqrt{48}=\sqrt{GPF}\sqrt{what\ it\ multiplies\ by}$:
$\sqrt{48}=\sqrt{16}\sqrt{3}$
Then, square root the GPF, leaving the other root alone.
$\sqrt{48}=\boxed{4\sqrt{3}}$

EXAMPLE Simplify. $\sqrt{-48}$

First, remove i.
$\sqrt{-48}=i\sqrt{48}$
*Next, find the greatest **perfect** factor.*

48
$1\cdot 48$
$2\cdot 24$
$3\cdot \boxed{16}$ ←GPF
$4\cdot 12$
$6\cdot 8$

Then, rewrite the root using the GPF:
$\sqrt{-48}=i\sqrt{16}\sqrt{3}$
Square root the GPF, leaving the other root alone. "i" goes between the number and the root.
$\sqrt{-48}=\boxed{4i\sqrt{3}}$

Simplify as much as possible.

1. $\sqrt{27}$	2. $\sqrt{-27}$	3. $\sqrt{500}$	4. $\sqrt{-500}$
5. $\sqrt{45}$	6. $\sqrt{-45}$	7. $\sqrt{12}$	8. $\sqrt{-12}$
9. $\sqrt{242}$	10. $\sqrt{-242}$	11. $\sqrt{192}$	12. $\sqrt{-192}$
13. $\sqrt{8}$	14. $\sqrt{-8}$	15. $\sqrt{20}$	16. $\sqrt{-20}$
17. $\sqrt{18}$	18. $\sqrt{-18}$	19. $\sqrt{288}$	20. $\sqrt{-288}$
21. $\sqrt{24}$	22. $\sqrt{-24}$	23. $\sqrt{72}$	24. $\sqrt{-72}$

Answers

1. $3\sqrt{3}$	2. $3i\sqrt{3}$	3. $10\sqrt{5}$	4. $10i\sqrt{5}$
5. $3\sqrt{5}$	6. $3i\sqrt{5}$	7. $2\sqrt{3}$	8. $2i\sqrt{3}$
9. $11\sqrt{2}$	10. $11i\sqrt{2}$	11. $8\sqrt{3}$	12. $8i\sqrt{3}$
13. $2\sqrt{2}$	14. $2i\sqrt{2}$	15. $2\sqrt{5}$	16. $2i\sqrt{5}$
17. $3\sqrt{2}$	18. $3i\sqrt{2}$	19. $12\sqrt{2}$	20. $12i\sqrt{2}$
21. $2\sqrt{6}$	22. $2i\sqrt{6}$	23. $6\sqrt{2}$	24. $6i\sqrt{2}$

When it comes to dividing square roots, it is important to remember that a root and a non-root cannot be combined in any way (not by adding, subtracting, multiplying or dividing). The only way to simplify a fraction involving a square root is to first simplify the root so that part of it is divisible.

EXAMPLE Simplify.

$$\frac{\sqrt{48}}{-8}$$

First, you need to simplify the root.

$$\frac{\sqrt{48}}{-8} = \frac{\sqrt{16}\sqrt{3}}{-8} = \frac{4\sqrt{3}}{-8}$$

Then, divide the top & bottom non-root values by the GCF:

$$\frac{(4 \div -4)\sqrt{3}}{(-8 \div -4)} = \frac{-1\sqrt{3}}{2} = \boxed{-\frac{\sqrt{3}}{2}}$$

EXAMPLE Simplify.

$$\frac{\sqrt{-48}}{2}$$

First, you need to simplify the root.

$$\frac{\sqrt{-48}}{2} = \frac{i\sqrt{16}\sqrt{3}}{2} = \frac{4i\sqrt{3}}{2}$$

Then, divide the top & bottom non-root values by the GCF:

$$\frac{(4 \div 2)i\sqrt{3}}{(2 \div 2)} = \frac{2i\sqrt{3}}{1} = \boxed{2i\sqrt{3}}$$

Simplify as much as possible.

25. $\frac{\sqrt{27}}{-9}$	26. $\frac{\sqrt{-27}}{3}$	27. $\frac{\sqrt{500}}{5}$	28. $\frac{\sqrt{-500}}{-20}$
29. $\frac{\sqrt{45}}{6}$	30. $\frac{\sqrt{-45}}{-9}$	31. $\frac{\sqrt{12}}{2}$	32. $\frac{\sqrt{-12}}{-2}$
33. $\frac{\sqrt{242}}{44}$	34. $\frac{\sqrt{-242}}{33}$	35. $\frac{\sqrt{192}}{8}$	36. $\frac{\sqrt{-192}}{64}$
37. $\frac{\sqrt{8}}{-4}$	38. $\frac{\sqrt{-8}}{-2}$	39. $\frac{\sqrt{20}}{10}$	40. $\frac{\sqrt{-20}}{-10}$
41. $\frac{\sqrt{18}}{3}$	42. $\frac{\sqrt{-18}}{6}$	43. $\frac{\sqrt{288}}{6}$	44. $\frac{\sqrt{-288}}{3}$
45. $\frac{\sqrt{24}}{4}$	46. $\frac{\sqrt{-24}}{8}$	47. $\frac{\sqrt{72}}{12}$	48. $\frac{\sqrt{-72}}{6}$

Answers

25. $-\frac{\sqrt{3}}{3}$	26. $i\sqrt{3}$	27. $2\sqrt{5}$	28. $-\frac{i\sqrt{5}}{2}$
29. $\frac{\sqrt{5}}{2}$	30. $-\frac{i\sqrt{5}}{3}$	31. $\sqrt{3}$	32. $-i\sqrt{3}$
33. $\frac{\sqrt{2}}{4}$	34. $\frac{i\sqrt{2}}{3}$	35. $\sqrt{3}$	36. $\frac{i\sqrt{3}}{8}$
37. $-\frac{\sqrt{2}}{2}$	38. $-i\sqrt{2}$	39. $\frac{\sqrt{5}}{5}$	40. $-\frac{i\sqrt{5}}{5}$
41. $\sqrt{2}$	42. $\frac{i\sqrt{2}}{2}$	43. $2\sqrt{2}$	44. $4i\sqrt{2}$
45. $\frac{\sqrt{6}}{2}$	46. $\frac{i\sqrt{6}}{4}$	47. $\frac{\sqrt{2}}{2}$	48. $i\sqrt{2}$

EXAMPLE Simplify.

$$\frac{-16 \pm \sqrt{48}}{-8}$$

First, you need to simplify the root.

$$\frac{-16 \pm \sqrt{48}}{-8} = \frac{-16 \pm \sqrt{16}\sqrt{3}}{-8} = \frac{-16 \pm 4\sqrt{3}}{-8}$$

Normally, the next step would be adding and subtracting the top, but you can't. So you have to deal with the division instead: split the problem into two fractions – the left and the right –, each with the same denominator.

$$\frac{-16 \pm 4\sqrt{3}}{-8} = \frac{-16}{-8} \pm \frac{4\sqrt{3}}{-8}$$

Then, on each fraction, divide the top & bottom non-root values by the GCF:

$$\frac{-16}{-8} \pm \frac{4\sqrt{3}}{-8} = \quad \frac{(-16 \div -8)}{(-8 \div -8)} \quad \pm \quad \frac{(4 \div -4)\sqrt{3}}{(-8 \div -4)}$$

$$= \frac{2}{1} \pm \frac{-1\sqrt{3}}{2} = 2 \pm -\frac{\sqrt{3}}{2} = \boxed{2 \mp \frac{\sqrt{3}}{2}}$$

EXAMPLE Simplify.

$$\frac{1 \pm \sqrt{-48}}{2}$$

$$\frac{1 \pm \sqrt{-48}}{2} = \frac{1 \pm i\sqrt{16}\sqrt{3}}{2} = \frac{1 \pm 4i\sqrt{3}}{2}$$

$$= \frac{1}{2} \pm \frac{4i\sqrt{3}}{2}$$

$$= \frac{1 \div 1}{2 \div 1} \pm \frac{(4 \div 2)i\sqrt{3}}{(2 \div 2)} = \frac{1}{2} \pm \frac{2i\sqrt{3}}{1} = \boxed{\frac{1}{2} \pm 2i\sqrt{3}}$$

Simplify as much as possible.

49. $\frac{18 \pm \sqrt{27}}{-9}$	50. $\frac{3 \pm \sqrt{-27}}{3}$	51. $\frac{12 \pm \sqrt{500}}{5}$	52. $\frac{-15 \pm \sqrt{-500}}{-20}$
53. $\frac{-4 \pm \sqrt{45}}{6}$	54. $\frac{12 \pm \sqrt{-45}}{-9}$	55. $\frac{6 \pm \sqrt{12}}{2}$	56. $\frac{-6 \pm \sqrt{-12}}{-2}$
57. $\frac{22 \pm \sqrt{242}}{44}$	58. $\frac{22 \pm \sqrt{-242}}{33}$	59. $\frac{-10 \pm \sqrt{192}}{8}$	60. $\frac{-32 \pm \sqrt{-192}}{64}$

Answers

49. $-2 \pm \frac{\sqrt{3}}{3}$	50. $1 \pm i\sqrt{3}$	51. $\frac{12}{5} \pm 2\sqrt{5}$	52. $\frac{3}{4} \pm \frac{i\sqrt{5}}{2}$
53. $-\frac{2}{3} \pm \frac{\sqrt{5}}{2}$	54. $-\frac{4}{3} \pm \frac{i\sqrt{5}}{3}$	55. $3 \pm \sqrt{3}$	56. $3 \pm i\sqrt{3}$
57. $\frac{1}{2} \pm \frac{\sqrt{2}}{4}$	58. $\frac{2}{3} \pm \frac{i\sqrt{2}}{3}$	59. $-\frac{5}{4} \pm \sqrt{3}$	60. $-\frac{1}{2} \pm \frac{i\sqrt{3}}{8}$