

## Quadratic Details Part 1

**EXAMPLE**

$$f(x) = 3x^2 - 30x + 33$$

**Vertex:** In standard form, you find the vertex with the formula  $x = \frac{-b}{2a}$ , then you plug in that x-value to find y.

$$x = \frac{-b}{2a} = \frac{-(-30)}{2(3)} = \frac{30}{6} = 5$$

$$x = 5$$

The axis of symmetry is the x-value.

$$y = 3x^2 - 30x + 12$$

$$y = 3(5)^2 - 30(5) + 12$$

$$y = 3(25) - 150 + 12$$

$$y = 75 - 150 + 12 = -63$$

$$y = -63$$

Graph is positive, so it faces Up.  
It has a minimum at the y-value.

$$\text{Vertex: } (5, -63)$$

**y-int:** The y-intercept is always the point where  $x = 0$ . In standard form,  $x = 0$  cancels out everything except  $c$ , so you know what the y-intercept is:  $(0, c)$ . If you are unsure, though, you can simply plug in  $x = 0$ .

$$x = 0, \text{ so } y \text{ is } c$$

$$y = 3(0)^2 - 30(0) + 33$$

$$y = 0 - 0 + 33 = 33$$

$$y\text{-int: } (0, 33)$$

**Roots:** In standard form (assuming you don't choose to convert to another form), you can determine the roots by using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(3)(33)}}{2(3)}$$

$$x = \frac{30 \pm \sqrt{900 - 396}}{6} = \frac{30 \pm \sqrt{504}}{6}$$

$$x = \frac{30 \pm \sqrt{36 \cdot 14}}{6} = \frac{30 \pm 6\sqrt{14}}{6}$$

$$x = \frac{30}{6} \pm \frac{6\sqrt{14}}{6} = 5 \pm \sqrt{14}$$

$$x = 5 \pm \sqrt{14}$$

Vertex	(5, -63)
Axis of Symm	$x = 5$
Max or Min?	Minimum at $y = -63$
y-int.	(0, 33)
x-int.'s/ roots/ zeros/ solutions	$x = 5 \pm \sqrt{14}$

**EXAMPLE**

$$g(x) = -2(x + 7)(x - 1)$$

**Vertex:** In factored form, you find the vertex with the formula  $x = \frac{r_1 + r_2}{2}$ , then you plug in that x-value to find y.

$$x = \frac{r_1 + r_2}{2} = \frac{(-7) + (1)}{2} = \frac{-6}{2}$$

$$x = -3$$

The axis of symmetry is the x-value.

$$y = -2(x + 7)(x - 1)$$

$$y = -2(-3 + 7)(-3 - 1)$$

$$y = -2(4)(-4)$$

$$y = 32$$

Graph is negative, so it faces down.  
It has a maximum at the y-value.

$$\text{Vertex: } (-3, 32)$$

**y-int:** The y-intercept is always the point where  $x = 0$ . So, plug in  $x = 0$ .

$$y = -2(0 + 7)(0 - 1)$$

$$y = -2(7)(-1) = 14 \quad (0, 14)$$

**Roots:** In factored form, the roots are given to you - they're the numbers in the parentheses with  $x$ . Don't forget to watch the signs!

$$-2(x + 7)(x - 1), \text{ so } x = -7 \text{ \& } x = 1$$

Vertex	(-3, 32)
Axis of Symm	$x = -3$
Max or Min?	Maximum at $y = 32$
y-int.	(0, 14)
x-int.'s/ roots/ zeros/ solutions	$x = -7$ $x = 1$

**EXAMPLE**

$$h(x) = 5(x - 3)^2 + 625$$

**Vertex:** In vertex form, the vertex is given to you - the x-value is inside the parentheses with  $x$  (don't forget to switch the sign), and the y-value is the number added to the end (keep the sign you have).

$$h(x) = 5(x - 3)^2 + 625$$

$$x = h \text{ (switch the sign)}$$

$$x = +3$$

The axis of symmetry is the x-value.

$$y = k \text{ (keep the sign)}$$

$$y = 625$$

Graph is positive, so it faces Up.  
It has a minimum at the y-value.

$$\text{Vertex: } (3, 625)$$

**y-int:** The y-intercept is always the point where  $x = 0$ . So, plug in  $x = 0$ .

$$y = 5(0 - 3)^2 + 625$$

$$y = 5(-3)^2 + 625 = 5(9) + 625$$

$$y = 45 + 625 = 670 \quad (0, 670)$$

**Roots:** The roots are always the points where  $y = 0$ . In vertex form, you simply plug in  $y = 0$  and use SADMEP to solve.

$$0 = 5(x - 3)^2 + 625$$

$$-625 = 5(x - 3)^2$$

$$-25 = (x - 3)^2$$

$$\pm\sqrt{25} = (x - 3)$$

$$\pm 5 = x - 3$$

$$3 \pm 5 = x$$

$$x = 3 \pm 5$$

Vertex	(3, 625)
Axis of Symm	$x = 3$
Max or Min?	Minimum at $y = 625$
y-int.	(0, 670)
x-int.'s/ roots/ zeros/ solutions	$x = 3 \pm 5$

**Determine the details of each quadratic.**

1.  $f(x) = -4x^2 + 16x - 8$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

2.  $g(x) = -3(x - 2)(x + 4)$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

3.  $h(x) = -(x - 1)^2 + 20$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

4.  $j(x) = 2x^2 - 12x + 16$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

5.  $k(x) = 5(x + 3)(x + 3)$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

6.  $m(x) = 2(x + 5)^2 + 32$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

7.  $n(x) = 3x^2 + 24x + 48$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

8.  $p(x) = -2(x - 5)(x - 1)$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

9.  $q(x) = -5(x - 2)^2 - 100$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

10.  $r(x) = -(x + 8)(x + 2)$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

11.  $t(x) = -3(x + 6)^2 + 12$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

12.  $v(x) = x^2 + 2x + 3$

Vertex	
Axis of Symm	
Max or Min?	
y-int.	
x-int.'s/ roots/ zeros/ solutions	

13. $f(x) = 4(x - 3)^2 - 72$	14. $g(x) = -5x^2 - 20x - 8$	15. $h(x) = 4(x)(x - 4)$
Vertex	Vertex	Vertex
Axis of Symm	Axis of Symm	Axis of Symm
Max or Min?	Max or Min?	Max or Min?
y-int.	y-int.	y-int.
x-int.'s/ roots/ zeros/ solutions	x-int.'s/ roots/ zeros/ solutions	x-int.'s/ roots/ zeros/ solutions

### Answers

1. V: (2, 8) AofS: $x = 2$ Max: $y = 8$ y-int: (0, -8) Roots: $x = 2 \pm \sqrt{2}$	2. V: (-1, 27) AofS: $x = -1$ Max: $y = 27$ y-int: (0, 24) Roots: $x = 2$ or $x = -4$	3. V: (1, 20) AofS: $x = 1$ Max: $y = 20$ y-int: (0, 19) Roots: $x = 1 \pm 2\sqrt{5}$	4. V: (3, -2) AofS: $x = 3$ Min: $y = -2$ y-int: (0, 16) Roots: $x = 4$ or $x = 2$	5. V: (-3, 0) AofS: $x = -3$ Min: $y = 0$ y-int: (0, 45) Roots: $x = -3$
6. V: (-5, 32) AofS: $x = -5$ Min: $y = 32$ y-int: (0, 82) Roots: $x = -5 \pm 4i$	7. V: (-4, 0) AofS: $x = -4$ Min: $y = 0$ y-int: (0, 48) Roots: $x = -4$	8. V: (3, 8) AofS: $x = 3$ Max: $y = 8$ y-int: (0, -10) Roots: $x = 5$ or $x = 1$	9. V: (2, -100) AofS: $x = 2$ Max: $y = -100$ y-int: (0, -120) Roots: $x = 2 \pm 2i\sqrt{5}$	10. V: (-5, 9) AofS: $x = -5$ Max: $y = 9$ y-int: (0, -16) Roots: $x = -8$ or $x = -2$
11. V: (-6, -12) AofS: $x = -6$ Max: $y = 12$ y-int: (0, -96) Roots: $x = -8$ or $x = -4$	12. V: (-1, 2) AofS: $x = -1$ Min: $y = 2$ y-int: (0, 3) Roots: $x = -1 \pm i\sqrt{2}$	13. V: (3, -72) AofS: $x = 3$ Min: $y = -72$ y-int: (0, -36) Roots: $x = 3 \pm 3\sqrt{2}$	14. V: (-2, 32) AofS: $x = -2$ Max: $y = 32$ y-int: (0, -8) Roots: $x = -2 \mp \frac{2\sqrt{15}}{5}$	15. V: (2, -16) AofS: $x = 2$ Min: $y = -16$ y-int: (0, 0) Roots: $x = 0$ or $x = 4$