Name: _____

Congruence, Similarity and the Triangle Sum Theorem (Part 1)

The triangle sum theorem is used to determine angle measures on a single triangle, but if you have proof that two triangles are congruent (entirely the same: ≅) or are similar (the same shape: ~), then you can also use the triangle sum theorem to determine angle measures based on the relationships between the two triangles.

CPCTC:
If the triangles are congruent (≅), then all the parts (sides & angles) must also be congruent (≅).

Example:

$\triangle ABC \cong \triangle DEF$	Given
$\angle A \cong \angle D,\quad \angle B \cong \angle E$ & $\angle C \cong \angle F$	**CPCTC**
$\overline{AB} \cong \overline{DE},\quad \overline{BC} \cong \overline{EF}$ & $\overline{AC} \cong \overline{DF}$	**CPCTC**

CASTC:
If the triangles are similar (~), then all of the angles (but NOT the sides) must be congruent (≅).

Example:

$\triangle ABC \sim \triangle DEF$	Given
$\angle A \cong \angle D,\quad \angle B \cong \angle E$ & $\angle C \cong \angle F$	**CASTC**

Definition of Congruence:
If figures are congruent then the measures of the matching figures are equal to each other, and vice versa. $\cong \leftrightarrow =$

Examples:

$m\angle A = m\angle B$	Given
$\angle A \cong \angle B$	**Def.** ≅

$\angle A \cong \angle B$	Given
$m\angle A = m\angle B$	**Def.** ≅

Example: $\triangle ABC \cong \triangle DEF. m\angle A = 47°,\ \&\ m\angle B = 46°$. **Determine the $m\angle F$**. Fill in the table as accurately as possible.

Statements	Reasons
$\triangle ABC \cong \triangle DEF$ $m\angle A = 47°$ & $m\angle B = 46°$	**Given**
$m\angle A + m\angle B + m\angle C = 180$	**△ Sum Thm**
$47 + 46 + m\angle C = 180$	**Subst.**
$93 + m\angle C = 180$	**Simp.**
$m\angle C = 87°$	**Subtr. Prop. =**
$\angle C \cong \angle F$	Use **CPCTC** to show that, because the triangles were marked ≅ **before**, each angle is ≅ to the angle it matches (look at the order of the triangle names).
$m\angle C = m\angle F$	Use **Definition of Congruence (≅)** to change the symbol from ≅ to =.
$87° = m\angle F$	**Subst.**

$\triangle ABC$:

Angle	Opp. Side
$m\angle B = 46°$	$\overline{AC}$
$m\angle A = 47°$	$\overline{BC}$
$m\angle C = 87°$	$\overline{AB}$

$\triangle DEF$:

Angle	Opp. Side
$m\angle E = 46°$	$\overline{DF}$
$m\angle D = 47°$	$\overline{EF}$
$m\angle F = 87°$	$\overline{DE}$

Example: $\triangle ABC \sim \triangle DFE. m\angle D = 18°,\ \&\ m\angle E = 109°$. Determine the $m\angle B$. Fill in the table as accurately as possible.

Statements	Reasons
$\triangle ABC \sim \triangle DFE$ $m\angle D = 18°$ & $m\angle E = 109°$	**Given**
$m\angle D + m\angle E + m\angle F = 180$	**△ Sum Thm**
$18 + 109 + m\angle F = 180$	**Subst.**
$127 + m\angle F = 180$	**Simp.**
$m\angle F = 53°$	**Subtr. Prop. =**
$\angle B \cong \angle F$	**CASTC**
$m\angle B = m\angle F$	**Def.** ≅
$m\angle B = 53°$	**Subst.**

$\triangle ABC$:

Angle	Opp. Side
$m\angle A = 18°$	$\overline{BC}$
$m\angle B = 53°$	$\overline{AC}$
$m\angle C = 109°$	$\overline{AB}$

$\triangle$:

Angle	Opp. Side
$m\angle D = 18°$	$\overline{FE}$
$m\angle F = 53°$	$\overline{DE}$
$m\angle E = 109°$	$\overline{DF}$

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1. $\triangle ABC \cong \triangle EFD. m\angle B = 50°,\ \& m\angle C = 96°.$ Determine the $m\angle E$. Fill in the table as accurately as possible.

Statements	Reasons

$\triangle ABC$:

Angle	Opp. Side

$\triangle EFD$:

Angle	Opp. Side

2. $\triangle ABC \sim \triangle DFE. m\angle A = 39°,\ \& m\angle B = 95°.$ Determine the $m\angle E$. Fill in the table as accurately as possible.

Statements	Reasons

$\triangle ABC$:

Angle	Opp. Side

$\triangle DFE$:

Angle	Opp. Side

3. $\triangle ABC \sim \triangle EFD. m\angle E = 36°,\ \& m\angle D = 63°.$ Determine the $m\angle$B. Fill in the table as accurately as possible.

Statements	Reasons

$\triangle ABC$:

Angle	Opp. Side

$\triangle EFD$:

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4. $\triangle ABC \cong \triangle FDE. m\angle D = 99°, \ \& \ m\angle F = 51°$. Determine the $m\angle C$. Fill in the table as accurately as possible.

Statements	Reasons

$\triangle ABC$:

Angle	Opp. Side

$\triangle FDE$:

5. $\triangle ABC \sim \triangle DEF. m\angle A = 89°, \ \& \ m\angle C = 32°$. Determine the $m\angle E$. Fill in the table as accurately as possible.

Statements	Reasons

$\triangle ABC$:

Angle	Opp. Side

$\triangle DEF$:

6. $\triangle ABC \cong \triangle FED. m\angle E = 31°, \ \& \ m\angle F = 41°$. Determine the $m\angle C$. Fill in the table as accurately as possible.

Statements	Reasons

$\triangle ABC$:

Angle	Opp. Side

$\triangle FED$:

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7. $\triangle ABC \sim \triangle EDF. m\angle F = 52°,\ \&\ m\angle E = 85°$. Determine the $m\angle B$. Fill in the table as accurately as possible.

Statements	Reasons

$\triangle ABC$:

Angle	Opp. Side

$\triangle EDF$:

8. $\triangle ABC \sim \triangle DEF.\ m\angle A = 56°,\ \&\ m\angle B = 44°$. Determine the $m\angle F$. Fill in the table as accurately as possible.

Statements	Reasons

$\triangle ABC$:

Angle	Opp. Side

$\triangle EDF$:

Congruence, Similarity & the Triangle Sum Theorem Part 1 Answers

1. $\triangle ABC$: $\triangle EFD$:

$m\angle A = 34°$	$\overline{BC}$	$m\angle E = 34°$	$\overline{FD}$
$m\angle B = 50°$	$\overline{AC}$	$m\angle F = 50°$	$\overline{ED}$
$m\angle C = 96°$	$\overline{AB}$	$m\angle D = 96°$	$\overline{EF}$

2. $\triangle ABC$: $\triangle DFE$:

$m\angle A = 39°$	$\overline{BC}$	$m\angle D = 39°$	$\overline{FE}$
$m\angle C = 46°$	$\overline{AB}$	$m\angle E = 46°$	$\overline{DF}$
$m\angle B = 95°$	$\overline{AC}$	$m\angle F = 95°$	$\overline{DE}$

3. $\triangle ABC$: $\triangle$EFD:

$m\angle A = 36°$	$\overline{BC}$	$m\angle E = 36°$	$\overline{FD}$
$m\angle C = 63°$	$\overline{AB}$	$m\angle D = 63°$	$\overline{EF}$
$m\angle B = 81°$	$\overline{AC}$	$m\angle F = 81°$	$\overline{ED}$

4. $\triangle ABC$: $\triangle FDE$:

$m\angle C = 30°$	$\overline{AB}$	$m\angle E = 30°$	$\overline{FD}$
$m\angle A = 51°$	$\overline{BC}$	$m\angle F = 51°$	$\overline{DE}$
$m\angle B = 99°$	$\overline{AC}$	$m\angle D = 99°$	$\overline{FE}$

5. $\triangle ABC$: $\triangle$DEF:

$m\angle C = 32°$	$\overline{AB}$	$m\angle F = 32°$	$\overline{DE}$
$m\angle B = 59°$	$\overline{AC}$	$m\angle E = 59°$	$\overline{DF}$
$m\angle A = 89°$	$\overline{BC}$	$m\angle D = 89°$	$\overline{EF}$

6. $\triangle ABC$: $\triangle FED$:

$m\angle B = 31°$	$\overline{AC}$	$m\angle E = 31°$	$\overline{FD}$
$m\angle A = 41°$	$\overline{BC}$	$m\angle F = 41°$	$\overline{ED}$
$m\angle C = 108°$	$\overline{AB}$	$m\angle D = 108°$	$\overline{FE}$

7. $\triangle ABC$: $\triangle EDF$:

$m\angle B = 43°$	$\overline{AC}$	$m\angle D = 43°$	$\overline{EF}$
$m\angle C = 52°$	$\overline{AB}$	$m\angle F = 52°$	$\overline{ED}$
$m\angle A = 85°$	$\overline{BC}$	$m\angle E = 85°$	$\overline{DF}$

8. $\triangle ABC$: $\triangle$DEF:

$m\angle B = 44°$	$\overline{AC}$	$m\angle E = 44°$	$\overline{DF}$
$m\angle A = 56°$	$\overline{BC}$	$m\angle D = 56°$	$\overline{EF}$
$m\angle C = 80°$	$\overline{AB}$	$m\angle F = 80°$	$\overline{DE}$