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## Congruence, Similarity and the Triangle Sum Theorem (Part 1)

The triangle sum theorem is used to determine angle measures on a single triangle, but if you have proof that two triangles are congruent (entirely the same:  $\cong$ ) or are similar (the same shape:  $\sim$ ), then you can also use the triangle sum theorem to determine angle measures based on the relationships between the two triangles.

<b>CPCTC:</b> If the triangles are congruent ( $\cong$ ), then all the parts (sides & angles) must also be congruent ( $\cong$ ). <b>Example:</b> $\triangle ABC \cong \triangle DEF$ Given	CASTC: If the triangles are similar (~), then all of the angles (but NOT the sides) must be congruent (≅). Example:	<b>Definition of Congruence:</b> If figures are congruent then the measures of the matching figures are equal to each other, and vice versa. $\cong \leftrightarrow =$ <b>Examples:</b>
		$m \angle A = m \angle B$ Given $\angle A \cong \angle B$ Given $\angle A \cong \angle B$ $Def. \cong$ $m \angle A = m \angle B$ $Def. \cong$
$\overline{AB} \cong \overline{DE},  \overline{BC} \cong \overline{EF} \& \\ \overline{AC} \cong \overline{DF} \qquad \qquad$	$\angle C \cong \angle F$	

**Example:**  $\triangle ABC \cong \triangle DEF. m \angle A = 47^{\circ}$ ,  $\& m \angle B = 46^{\circ}$ . **Determine the**  $m \angle F$ . Fill in the table as accurately as possible.

Statements	Reasons
$\triangle ABC \cong \triangle DEF$ $m \angle A = 47^{\circ} \& m \angle B = 46^{\circ}$	Given
$m \angle A + m \angle B + m \angle C = 180$	<u>∧ Sum Thm</u>
$47 + 46 + m \angle C = 180$	Subst.
$93 + m \angle C = 180$	<u>Simp.</u>
$m \angle C = 87^{\circ}$	<u>Subtr. Prop. =</u>
$\angle C \cong \angle F$	Use <u>CPCTC</u> to show that, because the triangles were marked $\cong$ <b>before</b> , each angle is $\cong$ to the angle it matches (look at the order of the triangle names).
$m \angle \mathcal{C} = m \angle F$	Use <b>Definition of Congruence</b> ( $\cong$ ) to change the symbol from $\cong$ to $=$ .
87 ° = $m \angle F$	Subst.

 $\triangle ABC:$ 

Angle	Opp. Side
$m \angle B = 46^{\circ}$	$\overline{AC}$
$m \angle A = 47^{\circ}$	BC
$m \angle C = 87^{\circ}$	$\overline{AB}$

$\triangle DEF:$	Opp. Side
$m \angle E = 46^{\circ}$	$\overline{DF}$
$m \angle D = 47^{\circ}$	$\overline{EF}$
$m \angle F = 87^{\circ}$	DE

**Example:**  $\triangle ABC \sim \triangle DFE. m \angle D = 18^{\circ}$ ,  $\& m \angle E = 109^{\circ}$ . Determine the  $m \angle B$ . Fill in the table as accurately as possible.

Statements	Reasons	]	
$\triangle ABC \sim \triangle DFE$		$\triangle ABC$ :	
$m \angle D = 18^{\circ} \& m \angle E = 109^{\circ}$	<u>Given</u>	Angle	Opp. Side
$m \angle D + m \angle E + m \angle F = 180$	<u> ∆ Sum Thm</u>	$m \angle A = 18^{\circ}$ $m \angle B = 53^{\circ}$	$\frac{\overline{BC}}{\overline{AC}}$
$18 + 109 + m \angle F = 180$	<u>Subst.</u>	$m \angle C = 109^{\circ}$	$\overline{\overline{AB}}$
$127 + m \angle F = 180$	<u>Simp.</u>	Δ:	
$m \angle F = 53^{\circ}$	Subtr. Prop. =	$\frac{Angle}{m \angle D = 18^{\circ}}$	Opp. Side FE
$\angle B \cong \angle F$	CASTC		
$m \angle B = m \angle F$	<u>Def. ≅</u>	$m \angle F = 53^{\circ}$ $m \angle E = 109^{\circ}$	$\frac{\overline{DE}}{\overline{DF}}$
$m \angle B = 53^{\circ}$	<u>Subst.</u>		

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Statements	Reasons	$\Delta ABC:$ Angle	Opp. Side
		Aligie	000.300
		$\triangle EFD:$	Opp. Side
$BC \sim \Delta DFE. m \angle A = 39^\circ, \& m \angle B = 95^\circ.$ D	etermine the $m \angle E$ . Fill in the	e table as accurately a	as possible.
Statements	Reasons	$\triangle ABC$ :	
Sutoments		Angle	Opp. Side
		$\triangle DFE$ :	
		Angle	Opp. Side
$C \sim \Delta EFD. m \angle E = 36^\circ, \& m \angle D = 63^\circ. D$	etermine the $m \neq B$ Fill in the	table as accurately a	as nossible
$0 - 2 - 1 - 1 - 1 - 30$ , $\alpha + 1 - 30 - 30$ .	etermine the <i>m2D</i> . This in the		is possible.
Statements	Reasons	$\triangle ABC:$	Opp. Side
		$\triangle EFD$ :	
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$= \Delta FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible $= \Delta FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible $= \Delta FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible $= \Delta FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible $= \Delta FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible $= \Delta FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible $= \Delta FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible $= \Delta FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible $= \Delta FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible $= \Delta FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible $= \Delta FED.$	Statements	Reasons	△ ABC: Angle Opp.
$ \frac{  }{  } = \Delta DEF. m \angle A = 89^{\circ}, \& m \angle C = 32^{\circ}. Determine the m \angle E. Fill in the table as accurately as possible  Statements  ABC:  Mode  ABC:  Mode  ADEF:  AD$			
$\sim \Delta DEF. m \angle A = 89^{\circ}, \& m \angle C = 32^{\circ}. Determine the m \angle E. Fill in the table as accurately as possible          Statements       Reasons       \triangle ABC: $			
$\sim \Delta DEF. m \angle A = 89^\circ, \& m \angle C = 32^\circ. Determine the m \angle E. Fill in the table as accurately as possible          Statements       Reasons       \triangle ABC: $			
$\sim \Delta DEF. m \angle A = 89^{\circ}, \& m \angle C = 32^{\circ}. Determine the m \angle E. Fill in the table as accurately as possible          Statements       Reasons       \triangle ABC: \square $			
Statements       Reasons $\triangle ABC:$ Aude       Opp.1         Image:       Opp.2         Im			$\Delta$ FDE:
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Statements       Reasons $\triangle ABC:$ Aude       Opp.1         Image:       Opp.2         Im			
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Statements       Reasons $\triangle ABC:$ $Aude$ $Out         Aude Out         Aude Out         Aude Out    $			
Statements       Reasons $\triangle ABC:$ Ande       Opp.1         Image:			
Statements       Reasons $\triangle ABC:$ Aude       Opp.1         Image:       Opp.2         Im	$\sim A DEE m (A - 80^\circ 8 m (C - 32^\circ D))$	etermine the $m \not\in F$ fill in the	a table as accurately as possible
$\cong \triangle FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ. Determine the m \angle C. Fill in the table as accurately as possible  Statements Angle Out in the table as accurately as possible in the table in th$	DEF. MZA = 07, $a mZC = 32$ . D	etermine the $m \geq L$ . Fin in the	
$F \cong \triangle FED. m \angle E = 31^\circ, \& m \angle F = 41^\circ.$ Determine the $m \angle C$ . Fill in the table as accurately as possible Statements $ABC: \qquad Our : Aragle & Our : Arag$	Statements	Reasons	
$\cong \triangle FED. m \angle E = 31^\circ$ , $\& m \angle F = 41^\circ$ . Determine the $m \angle C$ . Fill in the table as accurately as possible         Statements       Reasons $\triangle ABC:$ $Angle$ Opp.5 $\square$ $\square$ $\square$ $\square$ $\square$ $\square$			Angle Opp.
$\cong \triangle FED. m \angle E = 31^{\circ}, \& m \angle F = 41^{\circ}. Determine the m \angle C. Fill in the table as accurately as possible          Statements       Reasons       \triangle ABC:         Angle       Opp.5   $			
$\cong \triangle FED. m \angle E = 31^{\circ}, \& m \angle F = 41^{\circ}. Determine the m \angle C. Fill in the table as accurately as possible          Statements       Reasons       \triangle ABC:         Angle       Opp.5   $			
$\cong \Delta FED. \ m \angle E = 31^{\circ}, \ \& \ m \angle F = 41^{\circ}. \ Determine the \ m \angle C. \ Fill in the table as accurately as possible          Statements       Reasons       \Delta \ ABC: Angle Opp.5 $			
Statements     Reasons     △ ABC:			$\triangle DEF$ :
Statements     Reasons     △ ABC:       Angle     Opp. 5			
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Statements     Reasons     △ ABC:       Angle     Opp. 5			
Statements     Reasons     △ ABC:       Angle     Opp. 5			
Statements     Reasons     △ ABC:       Angle     Opp. 5	$\Gamma \sim A EED m / E = 21^{\circ} R m / E = 41^{\circ} D$	otorming the m/C Fill in the	table as accurately as possible
Angle Opp. 5	$Z = \Delta T E D \cdot m Z E - S I , \& m Z T - H I \cdot D$	the the matche matche the the the the the the the the the t	e table as accurately as possible
	Statements	Reasons	$\triangle ABC:$
Δ FED: Δ FED: Δ ΓΕΔ.			Angle Opp.
Δ FED:			
△ FED:			
Δ FED:			
			$\triangle$ FED:

Name:							
7. △ <i>ABC</i> ~△ <i>EDF</i> . <i>m</i> ∠ <i>F</i> = 52°, & <i>m</i> ∠ <i>E</i> = 85°. Determine the <i>m</i> ∠ <i>B</i> . Fill in the table as accurately as possible.							
	Sta	tements		Reasons	Δ A		
						Angle	Opp. Side
					$\triangle E$	DF:	
8. $\triangle ABC \sim \triangle DE$	$EF. m \angle A =$	56°, $\& m \angle B = 44$	°. Determine	the $m \angle F$ . Fill in t	he table as a	ccurately as poss	sible.
	Sta	tements		Reasons	∆ A	BC:	
						Angle	Opp. Side
					$\triangle E$	DF:	
	<u>Co</u>	<u>ngruence, Similar</u>	<u>ity &amp; the Tria</u> r	<u>ıgle Sum Theoren</u>	<u>n Part 1 Ansv</u>	<u>wers</u>	
1. <i>△ ABC</i> :		$\triangle$ EFD:		2. <i>△ ABC</i> :		$\triangle DFE$ :	
$m \angle A = 34^{\circ}$	BC	$m \angle E = 34^{\circ}$	$\overline{FD}$	$m \angle A = 39^{\circ}$	$\overline{BC}$	$m \angle D = 39^{\circ}$	$\overline{FE}$
$m \angle B = 50^{\circ}$	$\frac{DC}{AC}$	$m \angle F = 50^{\circ}$	$\overline{ED}$	$m \angle C = 46^{\circ}$	$\overline{AB}$	$m \angle E = 46^{\circ}$	$\overline{DF}$
$m \angle C = 96^{\circ}$	$\overline{\overline{AB}}$	$m \angle D = 96^{\circ}$	ĒF	$m \ge B = 95^{\circ}$	$\overline{AC}$	$m \angle F = 95^{\circ}$	$\overline{DE}$
• •	- 12						
3. <i>△ ABC</i> :		△EFD:		4. $\triangle ABC$ :		$\triangle$ <i>FDE</i> :	
$m \angle A = 36^{\circ}$	BC	$m \angle E = 36^{\circ}$	$\overline{FD}$	$m \angle C = 30^{\circ}$	$\overline{AB}$	$m \angle E = 30^{\circ}$	$\overline{FD}$
$m \angle C = 63^{\circ}$	$\overline{AB}$	$m \angle D = 63^{\circ}$	$\overline{EF}$	$m \angle A = 51^{\circ}$	$\overline{BC}$	$m \angle F = 51^{\circ}$	$\overline{DE}$
$m \angle B = 81^{\circ}$	$\overline{AC}$	$m \angle F = 81^{\circ}$	$\overline{ED}$	$m \angle B = 99^{\circ}$	$\overline{AC}$	$m \angle D = 99^{\circ}$	$\overline{FE}$
5. <i>△ ABC</i> :		△DEF:		6. △ <i>ABC</i> :		$\triangle$ FED:	
$m \angle C = 32^{\circ}$	$\overline{AB}$	$m \angle F = 32^{\circ}$	$\overline{DE}$	$m \angle B = 31^{\circ}$	$\overline{AC}$	$m \angle E = 31^{\circ}$	$\overline{FD}$

 $m \angle B = 31^{\circ}$  $m \angle A = 41^{\circ}$  $m \angle B = 59^{\circ}$ ĀĈ  $m \angle E = 59^{\circ}$  $\overline{DF}$ BC  $m \angle F = 41^{\circ}$  $\overline{ED}$  $m \angle A = 89^{\circ}$  $m \angle D = 89^{\circ}$  $m \angle C = 108^\circ$  $m \angle D = 108^\circ$ ВC  $\overline{EF}$  $\overline{AB}$  $\overline{FE}$ 7. *△ ABC*:  $\triangle EDF$ : 8. *△ ABC*: △DEF:  $\overline{EF}$  $\overline{AC}$ ĀĈ  $\overline{DF}$  $m \angle B = 43^{\circ}$  $m \angle D = 43^{\circ}$  $m \angle B = 44^{\circ}$  $m \angle E = 44^\circ$ BC  $m \angle C = 52^{\circ}$  $\overline{AB}$  $\overline{ED}$  $\overline{EF}$  $m \angle F = 52^\circ$  $m \angle A = 56^{\circ}$  $m \angle D = 56^{\circ}$  $m \angle A = 85^{\circ}$ BC  $\overline{DF}$  $\overline{AB}$  $\overline{DE}$  $m \angle E = 85^\circ$  $m \angle C = 80^\circ$  $m \angle F = 80^\circ$ 

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