Name: ______________________________

Congruent and Similar Triangles

<u>To prove triangles are **congruent**</u>, use one of the five properties below.

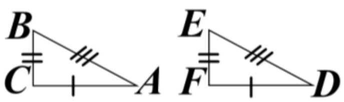
Remember, for congruent, ANGLE pairs have to be congruent (have the same measure),

and SIDE pairs have to be congruent

SSS

When all three sides on the first triangle match all three sides on the second, use SSS.

Given: $\overline{BC} \cong \overline{EF}, \overline{AC} \cong \overline{DF}, \overline{AB} \cong \overline{DE}$



Prove: $\triangle ABC \cong \triangle DEF$

$\angle A$	$\overline{BC}$
$\angle B$	$\overline{AC}$
$\angle C$	$\overline{AB}$

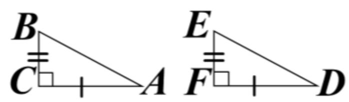
$\angle D$	$\overline{EF}$
$\angle E$	$\overline{DF}$
$\angle F$	$\overline{DE}$

Statements	Reasons
$\overline{BC} \cong \overline{EF}$	Given
$\overline{AC} \cong \overline{DF}$	Given
$\overline{AB} \cong \overline{DE}$	Given
$\triangle \boldsymbol{ABC} \cong \triangle \boldsymbol{DEF}$	**SSS**

SAS

When two sides on the first triangle match two sides on the second <u>and</u> the angle in between (not opposite) matches, as well, use SAS.

Given: $\overline{BC} \cong \overline{EF}, \overline{AC} \cong \overline{DF}$,
$m\angle C = 90° \ \& \ m\angle F = 90°$



Prove: $\triangle ABC \cong \triangle DEF$

$\angle A$	$\overline{BC}$
$\angle B$	$\overline{AC}$
$m\angle C = 90°$	$\overline{AB}$

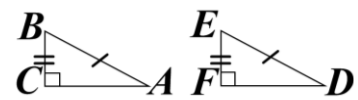
$\angle D$	$\overline{EF}$
$\angle E$	$\overline{DF}$
$\angle F = 90°$	$\overline{DE}$

Statements	Reasons
$m\angle C = 90°$	Given
$m\angle F = 90°$	Given
$m\angle C = m\angle F$	Subst.
$\angle C \cong \angle F$	Def. $\cong$
$\overline{BC} \cong \overline{EF}$	Given
$\overline{AC} \cong \overline{DF}$	Given
$\triangle \boldsymbol{ABC} \cong \triangle \boldsymbol{DEF}$	**SAS**

HL (SSA with 90°)

When two sides on the first triangle match two sides on the second <u>and</u> the angle opposite is 90° on both triangles, use HL.

Given: $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}$,
$m\angle C = 90° \ \& \ m\angle F = 90°$



Prove: $\triangle ABC \cong \triangle DEF$

$\angle A$	$\overline{BC}$
$\angle B$	$\overline{AC}$
$m\angle C = 90°$	$\overline{AB}$

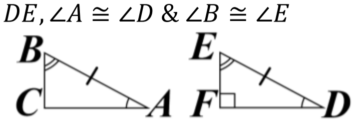
$\angle D$	$\overline{EF}$
$\angle E$	$\overline{DF}$
$\angle F = 90°$	$\overline{DE}$

Statements	Reasons
$m\angle C = 90°$	Given
$m\angle F = 90°$	Given
$m\angle C = m\angle F$	Subst.
$\angle C \cong \angle F$	Def. $\cong$
$\overline{AB} \cong \overline{DE}$	Given
$\overline{AC} \cong \overline{DF}$	Given
$\triangle \boldsymbol{ABC} \cong \triangle \boldsymbol{DEF}$	**HL**

ASA

When two angles on the first triangle match two angles on the second <u>and</u> the side in between (not opposite) matches, as well, use ASA.

Given: $\overline{AB} \cong \overline{DE}, \angle A \cong \angle D \ \& \ \angle B \cong \angle E$



Prove: $\triangle ABC \cong \triangle DEF$

$\angle A$	$\overline{BC}$
$\angle B$	$\overline{AC}$
$\angle C$	$\overline{AB}$

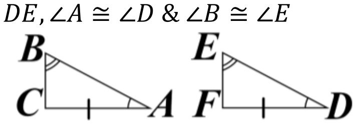
$\angle D$	$\overline{EF}$
$\angle E$	$\overline{DF}$
$\angle F$	$\overline{DE}$

Statements	Reasons
$\overline{AB} \cong \overline{DE}$	Given
$\angle A \cong \angle D$	Given
$\angle B \cong \angle E$	Given
$\triangle \boldsymbol{ABC} \cong \triangle \boldsymbol{DEF}$	**ASA**

AAS

When two angles on the first triangle match two angles on the second <u>and</u> the opposite side (of one of the matching angle sets) on both matches, use AAS.

Given: $\overline{AB} \cong \overline{DE}, \angle A \cong \angle D \ \& \ \angle B \cong \angle E$



Prove: $\triangle ABC \cong \triangle DEF$

$\angle A$	$\overline{BC}$
$\angle B$	$\overline{AC}$
$\angle C$	$\overline{AB}$

$\angle D$	$\overline{EF}$
$\angle E$	$\overline{DF}$
$\angle F$	$\overline{DE}$

Statements	Reasons
$\overline{AC} \cong \overline{DF}$	Given
$\angle A \cong \angle D$	Given
$\angle B \cong \angle E$	Given
$\triangle \boldsymbol{ABC} \cong \triangle \boldsymbol{DEF}$	**AAS**

Name: ____________________

To prove triangles are **similar**, use one of the five properties below.

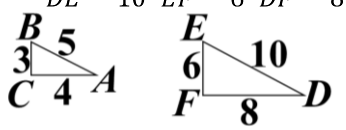
Remember, for similar, ANGLE pairs have to be congruent (have the same measure),
but SIDE pairs have to create equal scale fractions

If the triangles are congruent, then they are automatically similar, as well!

SSS

When all three sides on the first triangle create equal fractions with all three sides on the second, use SSS.

Given: $\frac{AB}{DE} = \frac{5}{10}, \frac{BC}{EF} = \frac{3}{6}, \frac{AC}{DF} = \frac{4}{8}$



Prove: $\triangle ABC \sim \triangle DEF$

$\angle A$	$\overline{BC}$
∠B	$\overline{AC}$
∠C	$\overline{AB}$

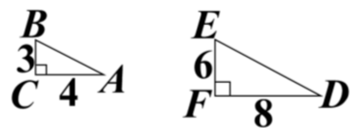
∠D	$\overline{EF}$
∠E	$\overline{DF}$
$\angle F$	$\overline{DE}$

Statements	Reasons
$\frac{AB}{DE} = \frac{5}{10}, \frac{BC}{EF} = \frac{3}{6}, \frac{AC}{DF} = \frac{4}{8}$	Given
$\frac{AB}{DE} = \frac{1}{2}, \frac{BC}{EF} = \frac{1}{2}, \frac{AC}{DF} = \frac{1}{2}$	Simp.
$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$	Subst. Prop. =
$\boldsymbol{\triangle ABC \sim \triangle DEF}$	**SSS**

SAS

When two sides on the first triangle create equal fractions with two sides on the second and the angle in between (not opposite) matches, as well, use SAS.

Given: $\frac{BC}{EF} = \frac{3}{6}, \frac{AC}{DF} = \frac{4}{8},$
$m\angle C = 90°$ & $m\angle F = 90°$



Prove: $\triangle ABC \sim \triangle DEF$

$\angle A$	$\overline{BC}$
∠B	$\overline{AC}$
$m\angle$C = 90°	$\overline{AB}$

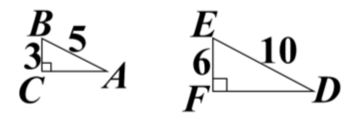
∠D	$\overline{EF}$
∠E	$\overline{DF}$
∠F = 90°	$\overline{DE}$

Statements	Reasons
$m\angle C = 90°$	Given
$m\angle F = 90°$	Given
$m\angle C = m\angle F$	Subst.
$\angle C \cong \angle F$	Def. ≅
$\frac{BC}{EF} = \frac{3}{6}, \frac{AC}{DF} = \frac{4}{8}$	Given
$\frac{BC}{EF} = \frac{1}{2}, \frac{AC}{DF} = \frac{1}{2}$	Simp.
$\frac{BC}{EF} = \frac{AC}{DF}$	Subst. Prop. =
$\boldsymbol{\triangle ABC \sim \triangle DEF}$	**SAS**

HL (SSA with 90°)

When two sides on the first triangle create equal fractions with two sides on the second and the angle opposite is 90° on both triangles, use HL.

Given: $\frac{AB}{DE} = \frac{5}{10}, \frac{BC}{EF} = \frac{3}{6},$
$m\angle C = 90°$ & $m\angle F = 90°$



Prove: $\triangle ABC \sim \triangle DEF$

$\angle A$	$\overline{BC}$
∠B	$\overline{AC}$
$m\angle$C = 90°	$\overline{AB}$

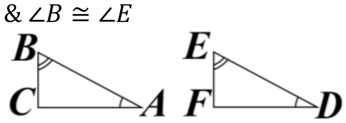
∠D	$\overline{EF}$
∠E	$\overline{DF}$
∠F = 90°	$\overline{DE}$

Statements	Reasons
$m\angle C = 90°$	Given
$m\angle F = 90°$	Given
$m\angle C = m\angle F$	Subst.
$\angle C \cong \angle F$	Def. ≅
$\frac{AB}{DE} = \frac{5}{10}, \frac{BC}{EF} = \frac{3}{6}$	Given
$\frac{AB}{DE} = \frac{1}{2}, \frac{BC}{EF} = \frac{1}{2}$	Simp.
$\frac{AB}{DE} = \frac{BC}{EF}$	Subst. Prop. =
$\boldsymbol{\triangle ABC \sim \triangle DEF}$	**HL**

AA

When two angles on the first triangle match two angles on the second, use AA.

Given: $\angle A \cong \angle D$ & $\angle B \cong \angle E$



Prove: $\triangle ABC \sim \triangle DEF$

$\angle A$	$\overline{BC}$
∠B	$\overline{AC}$
∠C	$\overline{AB}$

∠D	$\overline{EF}$
∠E	$\overline{DF}$
$\angle F$	$\overline{DE}$

Statements	Reasons
$\angle A \cong \angle D$	Given
$\angle B \cong \angle E$	Given
$\boldsymbol{\triangle ABC \sim \triangle DEF}$	**AA**

Name: ______________________________

Congruent and Similar Triangles

For each triangle, create a small-medium-large table (if you do not know the size of the angles, then put the angles of the first triangle in any order, then match that order for the second triangle). Then, determine if the triangles are congruent & similar, only similar or neither and by what property.

EXAMPLE:
On $\triangle LMN \;\; \triangle PQR$, $\angle L \cong \angle P$, $\angle M \cong \angle Q$, $LM = 8$ & $PQ = 24$. Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides
$\angle L$ $(\cong \angle P)$	$\overline{MN}$
$\angle M$ $(\cong \angle Q)$	$\overline{LN}$
$\angle N$	$LM = 8$

Angles	Opp. Sides
$(\angle L \cong)\angle P$	$\overline{QR}$
$(\angle M \cong)\angle Q$	$\overline{PR}$
$\angle R$	$PQ = 24$

There are 2 congruent angle pairs, which means we have AA. AA is used to prove similarity, but *is not enough* for congruence.

$\triangle LMN$ <u>$\sim$</u> $\triangle PQR$ by **<u>AA</u>**

EXAMPLE:
On $\triangle LMN \;\; \triangle PQR$, $m\angle N = 70°$, $m\angle R = 70°$, $LN = 8$, $MN = 7$, $PR = 24$ & $QR = 21$. Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides
$\angle L$	$MN = 7$
$\angle M$	$LN = 8$
$m\angle N = 70°$	$\overline{LM}$

Angles	Opp. Sides
$\angle P$	$QR = 21$
$\angle Q$	$PR = 24$
$m\angle R = 70°$	$\overline{PQ}$

There is only 1 congruent angle pair, which is not enough for congruence or for similarity. If we want to prove similarity, then we need to use the sides to create scale fractions $\left(\frac{small}{small} \& \frac{large}{large}\right)$ to see if they are the same.

$$\frac{MN}{QR} = \frac{7 \div 7}{21 \div 7} = \frac{1}{3} \qquad \frac{LM}{PQ} = \frac{8 \div 8}{24 \div 8} = \frac{1}{3}$$

The side fractions are the same, so we have 2 side pairs and 1 angle pair that is not opposite (SAS).

Angles	Opp. Sides
$\angle L$	$MN = 7$
$\angle M$	$LN = 8$
$m\angle N = 70°$	$\overline{LM}$

Angles	Opp. Sides
$\angle P$	$QR = 21$
$\angle Q$	$PR = 24$
$m\angle R = 70°$	$\overline{PQ}$

$\triangle LMN$ <u>$\sim$</u> $\triangle PQR$ by **<u>SAS</u>**

1. On $\triangle BCD$ & $\triangle EFG$, $\angle C \cong \angle F$, $\overline{CD} \cong \overline{FG}$ & $\overline{BC} \cong \overline{EF}$. Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides

Angles	Opp. Sides

$\triangle$ ____ $\triangle$ by _____

2. On $\triangle BCD$ & $\triangle EFG$, $m\angle B = 90°$, $m\angle E = 90°$, $m\angle D = 20°$, $m\angle G = 20°$ & $\overline{EG} \cong \overline{BD}$. Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides

Angles	Opp. Sides

$\triangle$ ____ $\triangle$ by _____

3. On $\triangle BCD$ & $\triangle EFG$, $BC = 4$, $BD = 6$, $CD = 8$, $FG = 12$, $EG = 9$ & $EF = 6$. Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides

Angles	Opp. Sides

$\triangle$ ____ $\triangle$ by _____

4. On $\triangle BCD$ & $\triangle EFG$, $m\angle C = 30°$, $m\angle F = 30°$, $m\angle D = 40°$, $m\angle G = 40°$ $\overline{BD} \cong \overline{EG}$. Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides

Angles	Opp. Sides

$\triangle$ ____ $\triangle$ by _____

Name: ______________________________

5. On $\triangle BCD$ & $\triangle EFG, \angle B \cong \angle E$ & $\angle D \cong \angle G$.
Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides

Angles	Opp. Sides

$\triangle$ ___ $\triangle$ by _____

6. On $\triangle BCD$ & $\triangle EFG, \angle D \cong \angle G$, $\overline{BC} \cong \overline{EF}$ & $\overline{CD} \cong \overline{FG}$.
Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides

Angles	Opp. Sides

$\triangle$ ___ $\triangle$ by _____

7. On $\triangle BCD$ & $\triangle EFG, CD = 18, BD = 6, FG = 15$, $EG = 5$ & $m\angle B = 90°, m\angle E = 90°$.
Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides

Angles	Opp. Sides

$\triangle$ ___ $\triangle$ by _____

8. On $\triangle BCD$ & $\triangle EFG, \frac{CD}{FG} = \frac{BD}{EG}$ & $\angle D \cong \angle G$.
Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides

Angles	Opp. Sides

$\triangle$ ___ $\triangle$ by _____

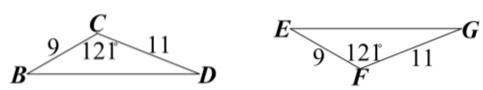
9. On $\triangle BCD$ & $\triangle EFG$ $\angle C \cong \angle F, \angle D \cong \angle G$, $\overline{CD} \cong \overline{FG}$,
Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides

Angles	Opp. Sides

$\triangle$ ___ $\triangle$ by _____

10.



Are the triangles congruent & similar ($\cong$ & $\sim$), similar ($\sim$), or neither? By what property?

Angles	Opp. Sides

Angles	Opp. Sides

$\triangle$ ___ $\triangle$ by _____

Congruent and Similar Triangles Answers

1. $\triangle BCD \cong \triangle EFG$ by SAS	2. $\triangle BCD \cong \triangle EFG$ by ASA	3. $\triangle BCD \sim \triangle EFG$ by SSS
4. $\triangle BCD \cong \triangle EFG$ by AAS	5. $\triangle BCD \sim \triangle EFG$ by AA	6. *Neither* – there is no evidence of congruence or similarity
7. $\triangle BCD \sim \triangle EFG$ by HL	8. $\triangle BCD \sim \triangle EFG$ by SAS	9. $\triangle BCD \cong \triangle EFG$ by ASA
10. $\triangle BCD \cong \triangle EFG$ by SAS		