Triangle Sum & Third Angles Theorems

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| Triangle Sum Theorem | Third Angles Theorem |
| The three angles of a triangle always add to equal 180˚. | SETUP: If two angles of a triangle are congruent to angles on another triangle,RESULT: Then the third angles on these two triangles will be congruent to each other as well. |
| Example:

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| Given:Prove: $m∠A+m∠B+m∠C=180˚$ |
| Statements | Reasons |
| $$m∠A+m∠B+m∠C=180˚$$ | Triangle Sum Thm. |

 | Example:

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| Given: $∠A≅∠F \& ∠B≅∠E$Prove: $∠C≅∠D $ |
| Statements | Reasons |
| $$∠A≅∠F \& ∠B≅∠E$$ | Given |
| $$∠C≅∠D$$ | Third Angles Thm. |

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**Fill in the proofs. Each proof was written using the triangles shown below.**

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| 1. |  | 2. |
| Given: $m∠G=53˚ \& m∠H=34˚ $ Prove: $87˚+m∠K=180˚$ |  | Given: $∠H≅∠P \& ∠K≅∠Q$ Prove: $m∠N=m∠G$ |
| Statements | Reasons |  | Statements | Reasons |
| $$m∠G+m∠H+m∠K=180˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠H≅∠P \& ∠K≅∠Q$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠G=53˚ \& m∠H=34˚ $$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠G≅∠N$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$53˚+34˚+m∠K=180$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠G=m∠N$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$87˚+m∠K=180$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠N=m∠G$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| 3. |  | 4. |
| Given: $∠K≅∠Q \& ∠G≅∠N$  Prove: $∠H≅∠P $ |  | Given: $m∠P=17˚$  Prove: $m∠N +m∠Q=163˚$ |
| Statements | Reasons |  | Statements | Reasons |
| $$∠K≅∠Q $$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠N+m∠P+m∠Q=180˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $∠G≅∠N$  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠N+17˚ +m∠Q=180˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠H≅∠P $$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠N +m∠Q=163˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| 5. |  | 6. |
| Given: $ ∠N≅∠E \& ∠L≅∠F$ Prove: $∠M≅∠D$ |  | Given: *See above image.* Prove: $m∠D+m∠E+m∠F=180˚$ |
| Statements | Reasons |  | Statements | Reasons |
| $$∠N≅∠E \& ∠L≅∠F$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠D+m∠E+m∠F=180˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠M≅∠D$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  |  |  |

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| 7. |  | 8. |
| Given:$∠L≅∠F \& ∠M≅∠D$ Prove: $m∠L+m∠M+m∠E=180˚$ |  | Given:$∠L≅∠F, ∠M≅∠D, \& \overbar{MN}≅\overbar{DE}$ Prove: $FE=LN$ |
| Statements | Reasons |  | Statements | Reasons |
| $$m∠L+m∠M+m∠N=180˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠L≅∠F, ∠M≅∠D, \&$$$$\overbar{MN}≅\overbar{DE}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠L≅∠F \& ∠M≅∠D$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△LNM≅ △FED$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠N≅∠E$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{LN}≅\overbar{FE}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠N=m∠E$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{FE}≅\overbar{LN}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠L+m∠M+m∠E=180$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$FE=LN$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

**Our focus this unit has been on proofs. However, it is just as important for you to be able to solve a problem (without the proof structure). For the following problems, use what you know to determine the answer.**

Complementary & Supplementary

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| 9. $∠ABC$ and $∠DEF$ are complementary. $m∠ABC=31°$. What is $m∠DEF$?(Complementary: \_\_\_\_\_\_ + \_\_\_\_\_\_ = 90$°$) | 10. $∠ABC$ and $∠DEF$ are supplementary. $m∠ABC=31°$. What is $m∠DEF$?(Supplementary: \_\_\_\_\_\_ + \_\_\_\_\_\_ = 180$°$) |
| 11. $∠LMN$ and $∠PQR$ are complementary. $m∠LMN=\left(2x+4\right)°$ & $m∠PQR=(7x-13)°$. What is $m∠LMN$? *Find x & plug it in.*(Complementary: \_\_\_\_\_\_ + \_\_\_\_\_\_ = 90$°$) | 12. $∠LMN$ and $∠PQR$ are supplementary. $m∠LMN=\left(2x+4\right)°$ & $m∠PQR=(7x-13)°$. What is $m∠LMN$? *Find x & plug it in.*(Supplementary: \_\_\_\_\_\_ + \_\_\_\_\_\_ = 180$°$) |

Vertical Angles

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| 13. $∠LMN$ & $∠PMQ$ are vertical angles. $m∠LMN=24°$. What is the $m∠PMQ$?(Vertical Angles: \_\_\_\_\_\_$ ≅$ \_\_\_\_\_\_, so \_\_\_\_\_\_ = \_\_\_\_\_\_) | 14. $∠BAC$ & $∠DAE$ are vertical angles. $m∠DAE=(4x+5)°$ & $m∠BAC=(6x-25)°$. What is $m∠DAE$?(Vertical Angles: \_\_\_\_\_\_$ ≅$ \_\_\_\_\_\_, so \_\_\_\_\_\_ = \_\_\_\_\_\_) |

Linear Pair Angles

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| 15. $∠LMN$ & $∠PMQ$ are linear pair angles. $m∠LMN=24°$. What is the $m∠PMQ$?(Linear Pair: \_\_\_\_\_\_ + \_\_\_\_\_\_ = 180$°$) | 16. $∠BAC$ & $∠DAE$ are vertical angles. $m∠DAE=(4x+5)°$ & $m∠BAC=(6x-25)°$. What is $m∠DAE$?(Linear Pair: \_\_\_\_\_\_ + \_\_\_\_\_\_ = 180$°$) |