Name: _____

Probabilities with "And" (Part 1)

When solving for a probability involving "and" or "both" or "3 times" or "4 digits," etc., what you're really solving is multiple probabilities combined. The **Counting Principle** tells us how to solve these types of problems. In basic terms, to combine multiple probabilities, you must multiply.

So, the sample size is created by multiplying the total from the 1st event times the total from the 2nd event (times the 3rd, and so on).

$$\textbf{"And"}\ Sample\ Size = \begin{pmatrix} 1st\ event's \\ total \end{pmatrix}\begin{pmatrix} 2nd\ event's \\ total \end{pmatrix}\begin{pmatrix} 3rd\ event's \\ total \end{pmatrix}(\dots)$$

Determining "and" probability works the same way:

$$\text{P("and")} = \left(\frac{1st\ want}{1st\ total}\right)\left(\frac{2nd\ want}{2nd\ total}\right)\left(\frac{3rd\ want}{3rd\ total}\right)(\dots)$$

or, more simply:

$$\text{P("and")} = (1st\ probability)(2nd\ probability)(3rd\ probability)(\dots)$$

"And" Using Categories

**EXAMPLE** There are 3 white shirts, 4 red shirts, 2 brown shirts, 3 black sweatshirts, 1 white sweatshirt, 2 pairs of white sneakers, 1 pair of black sneakers, 1 pair of sandals and 1 pair of dress shoes in the closet. If you randomly select 1 shirt, 1 sweatshirt, and 1 pair of shoes, what is the sample size for the combined probability? What is the probability that you will select a white shirt, black sweatshirt and a pair of white sneakers?

| Category | Shirts | | | Sweatshirts | | Shoes | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sample Space | White | Red | Brown | Black | White | White Sneakers | Black Sneakers | Sandals | Dress shoes |
| Amount | 3 | 4 | 2 | 3 | 1 | 2 | 1 | 1 | 1 |
| Sample Size | 9 shirts | 9 shirts | 9 shirts | 4 Sw-shirts | 4 Sw-shirts | 5 shoes | 5 shoes | 5 shoes | 5 shoes |
| Probability of each event | $\frac{3}{9}=\frac{1}{3}$ | $\frac{4}{9}$ | $\frac{2}{9}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{2}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ |

a. Combined Sample Size:

1st event: 9 shirts  2nd event: 4 Sweatshirts  3rd event: 5 shoes

$$Combined\ Sample\ Size = (9)(4)(5) = (36)(5) = \boxed{180}$$

b. $P(W\ Shirt\ \&\ B\ Sweatshirt\ \&\ W\ Sneakers) = P(W\ shirt)\cdot P(B\ Sweatshirt)\cdot P(W\ Sneakers)$

$$P(W\ Shirt\ \&\ B\ Sweatshirt\ \&\ W\ Sneakers) = \left(\frac{3}{9}\right)\left(\frac{3}{4}\right)\left(\frac{2}{5}\right) = \frac{(3)(3)(2)}{(9)(4)(5)} = \frac{18}{180}\dots reduce\ by\ 18\dots \boxed{\frac{1}{10}}$$

*(If you reduced the original probabilities first, you would get the same answer)*

$$P(W\ Shirt\ \&\ B\ Sweatshirt\ \&\ W\ Sneakers) = \left(\frac{1}{3}\right)\left(\frac{3}{4}\right)\left(\frac{2}{5}\right) = \frac{(1)(3)(2)}{(3)(4)(5)} = \frac{6}{60}\dots reduce\ by\ 6\dots \boxed{\frac{1}{10}}$$

**EXAMPLE** There are 2 guys and 1 girl running for club treasurer, 3 girls and 2 guys running for vice president and 4 guys and 4 girls running for club president. If you randomly select 1 treasurer, 1 vice president, and 1 president, what is the sample size for the combined probability? What is the probability that all three officers will be male?

| Category | Treasurer | | Vice President | | President | |
|---|---|---|---|---|---|---|
| Sample Space | Male | Female | Male | Female | Male | Female |
| Amount | 2 | 1 | 2 | 3 | 4 | 4 |
| Sample Size | 3 for treasurer | 3 for treasurer | 5 for VP | 5 for VP | 8 for President | 8 for President |
| Probability of each event | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{2}{5}$ | $\frac{3}{5}$ | $\frac{4}{8}=\frac{1}{2}$ | $\frac{4}{8}=\frac{1}{2}$ |

a. Combined Sample Size: $(3\ treasurers)(5\ VPs)(8\ Presidents) = (3)(5)(8) = \boxed{120}$

b. $P(Male\ Treasurer\ \&\ Male\ VP\ \&\ Male\ President) = P(M.Treasurer)\cdot P(M.VP)\cdot P(M.President)$

$$= \left(\frac{2}{3}\right)\left(\frac{2}{5}\right)\left(\frac{1}{2}\right) = \frac{(2)(2)(1)}{(3)(5)(2)} = \frac{4}{30} \ldots reduce \ldots \boxed{\frac{2}{15}}$$

1. There are 2 guys and 1 girl running for club treasurer, 3 girls and 2 guys running for vice president and 4 guys and 4 girls running for club president. If you randomly select 1 treasurer, 1 vice president, and 1 president, what is the sample size for the combined probability? What is the probability of selecting a male treasurer, female vice president, and male president?

| Category | Treasurer | | Vice President | | President | |
|---|---|---|---|---|---|---|
| Sample Space | Male | Female | Male | Female | Male | Female |
| Amount | | | | | | |
| Sample Size | | | | | | |
| Probability of each event | | | | | | |

a. Combined Sample Size:

b. $P($ $) =$

2. There are 2 guys and 1 girl running for club treasurer, 3 girls and 2 guys running for vice president and 4 guys and 4 girls running for club president. If you randomly select 1 treasurer, 1 vice president, and 1 president, what is the sample size for the combined probability? What is the probability that the treasurer will be female and both the vice president and president will be male?

| Category | Treasurer | | Vice President | | President | |
|---|---|---|---|---|---|---|
| Sample Space | Male | Female | Male | Female | Male | Female |
| Amount | | | | | | |
| Sample Size | | | | | | |
| Probability of each event | | | | | | |

a. Combined Sample Size:

b. $P($ $) =$

3. There are 12 baseballs & 3 tennis balls in the first bag, and 5 baseball jerseys, 3 football jerseys, and 2 plain shirts in the second bag. If you randomly select 1 item from each bag, what is the sample size for the combined probability? What is the probability of selecting a tennis ball and a plain shirt?

| Category | 1st Bag | | 2nd Bag | | |
|---|---|---|---|---|---|
| Sample Space | | | | | |
| Amount | | | | | |
| Sample Size | | | | | |
| Probability of each event | | | | | |

a. Combined Sample Size:

b. $P($ $) =$

4. There are 8 shirts in a drawer: 3 are blue, 2 are gray, 1 is black and the rest are purple. In another drawer, there are 4 pairs of pants: 2 pair are jeans, 1 pair are cargo pants, and 1 pair are slacks. If you randomly select 1 shirt and 1 pair of pants, what is the sample size for the combined probability? What is the probability of selecting a gray shirt and a pair of jeans?

| Category | | | | | | | |
|---|---|---|---|---|---|---|---|
| Sample Space | | | | | | | |
| Amount | | | | | | | |
| Sample Size | | | | | | | |
| Probability of each event | | | | | | | |

a. Combined Sample Size:

b. $P($ $) =$

5. There are 3 orange cats, 2 black cats, 1 gray dog, and 3 spotted dogs. If you randomly select 1 cat and 1 dog, what is the sample size for the combined probability? What is the probability of selecting an orange cat and a spotted dog?

| Category | | | | |
|---|---|---|---|---|
| Sample Space | | | | |
| Amount | | | | |
| Sample Size | | | | |
| Probability of each event | | | | |

a. Combined Sample Size:

b. $P($ $) =$

Basics of Probability with "And" (Part 1)
Answers

| | |
|---|---|
| 1.<br>$Combined\ Sample\ Space = \boxed{120}$<br>$P(Male\ T\ \&\ Female\ VP\ \&\ Male\ P) = \frac{24}{120} \ldots reduce \ldots$<br>$P(Male\ T\ \&\ Female\ VP\ \&\ Male\ P) = \boxed{\frac{1}{5}}$ | 2.<br>$Combined\ Sample\ Space = \boxed{120}$<br>$P(Female\ T\ \&\ Male\ VP\ \&\ Male\ P) = \frac{8}{120} \ldots reduce \ldots$<br>$P(Female\ T\ \&\ Male\ VP\ \&\ Male\ P) = \boxed{\frac{1}{15}}$ |
| 3.<br>$Combined\ Sample\ Space = \boxed{150}$<br>$P(Tennis\ Ball\ \&\ Plain\ Shirt) = \frac{6}{150} \ldots reduce \ldots$<br>$P(Tennis\ Ball\ \&\ Plain\ Shirt) = \boxed{\frac{1}{25}}$ | 4.<br>$Combined\ Sample\ Space = \boxed{32}$<br>$P(Gray\ Shirt\ \&\ Jeans) = \frac{4}{32} \ldots reduce \ldots$<br>$P(Gray\ Shirt\ \&\ Jeans) = \boxed{\frac{1}{8}}$ |
| 5.<br>$Combined\ Sample\ Space = \boxed{20}$<br>$P(Orange\ Cat\ \&\ Spotted\ Dog) = \boxed{\frac{9}{20}}$ | |