

Simplifying Radicals

We've worked with radicals in equations and we've done some basic simplification, but if you want to be able to handle problems involving radicals, there are a few other skills you need. First, you need to be able to take apart and put back together a radical using multiplication (product property) or division (quotient property). This process is like working with distribution-you can factor a radical out so that both products are using it together, or you can split it up so that each product gets its own radical.

Example	Your Turn	Still Your Turn
$\sqrt{18}\sqrt{2} = \sqrt{36}$ $= 6$ <i>You want a perfect square & $18 \times 2 = 36$</i>	1. $\sqrt{10}\sqrt{40}$	2. $\sqrt{3}\sqrt{75}$
$\sqrt{3600} = \sqrt{36}\sqrt{100}$ $= 6 \times 10$ $= 60$ <i>Use perfect square factors: 36 & 100 to simplify.</i>	3. $\sqrt{441}$	4. $\sqrt{576}$
$\frac{\sqrt{24}}{\sqrt{6}} = \sqrt{\frac{24}{6}}$ $= \sqrt{4}$ $= 2$ <i>You want a perfect square & $24 \div 6 = 4$</i>	5. $\frac{\sqrt{45}}{\sqrt{5}}$	6. $\frac{\sqrt{72}}{\sqrt{2}}$
$\sqrt{\frac{400}{9}} = \frac{\sqrt{400}}{\sqrt{9}}$ $= \frac{20}{3}$ <i>You want perfect squares. 400 & 9 are perfect, split up the radical and simplify.</i>	7. $\sqrt{\frac{49}{4}}$	8. $\sqrt{\frac{81}{36}}$

Another skill you need when working with radicals is the ability to "rationalize the denominator." All that means is that you need to be able to get a radical out of the bottom of a fraction. To do this, you must multiply the top and the bottom by the radical that you want to get rid of. Remember, though, just like with any fraction, if you can reduce the fraction, by canceling a common factor, you must. However, you have to be cautious. A radical can only reduce with another radical - normal numbers need not apply.

Tip: There is a shortcut that I like to use to solve these problems. If you do the math to rationalize the denominator, you will always get the same result: the radical with its radicand (the number inside) will end up multiplied on top of the fraction, and the bottom will just be the radicand (the number inside) - without that pesky radical.

$\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}}$ $= \frac{\sqrt{3}}{3}$ <i>Multiply the top and bottom by the radical in the bottom: $\sqrt{3}$</i> <i>Simplify.</i>	9. $\frac{1}{\sqrt{7}}$	10. $\frac{1}{\sqrt{5}}$
$\frac{1}{\sqrt{13}} \rightarrow \frac{\sqrt{13}}{13}$ <i>Trick: leave the radicand (number inside) on bottom, put the radical on top.</i>	11. $\frac{1}{\sqrt{15}}$	12. $\frac{1}{\sqrt{21}}$
$\frac{6}{\sqrt{12}} = \frac{6\sqrt{12}}{12}$ $= \frac{\sqrt{12}}{2}$ <i>Leave the radicand on bottom & multiply the top by the radical.</i> <i>Reduce by 6: $\frac{6}{12} = \frac{1}{2}$</i>	13. $\frac{5}{\sqrt{10}}$	14. $\frac{18}{\sqrt{3}}$

$\frac{4\sqrt{2}}{\sqrt{5}} = \frac{4\sqrt{2}\sqrt{5}}{5}$ $= \frac{4\sqrt{10}}{5}$	<p>15. $\frac{3\sqrt{7}}{\sqrt{2}}$</p>	<p>16. $\frac{9\sqrt{3}}{\sqrt{7}}$</p>
$\frac{8\sqrt{15}}{\sqrt{6}} = \frac{8\sqrt{5}}{\sqrt{2}}$ <p>Reduce the radicals by 3: $\frac{\sqrt{15}}{\sqrt{6}} = \frac{\sqrt{5}}{\sqrt{2}}$</p> $= \frac{8\sqrt{5}\sqrt{2}}{2}$ $= 4\sqrt{10}$ <p>Leave the radicand on the bottom & multiply the top by the radical. $\sqrt{5}\sqrt{2} = \sqrt{10}$</p>	<p>17. $\frac{14\sqrt{10}}{\sqrt{35}}$</p>	<p>18. $\frac{24\sqrt{40}}{\sqrt{24}}$</p>

When it comes to combining like terms, working with radicals is like working with a variable. If the radicands (the number in the radical, not the coefficient - the number in front) are the same, then you can add the terms. Careful, though - the radicand will not change, just like x doesn't change when you add it.

$3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$ <p>They have the same radicand - add 'em! 3+2=5</p>	<p>19. $9\sqrt{7} - 6\sqrt{7}$</p>	<p>20. $4\sqrt{2} + 8\sqrt{2}$</p>
$5\sqrt{3} - 2\sqrt{5} + 3\sqrt{5} - 6\sqrt{3}$ $5\sqrt{3} - 6\sqrt{3} - 2\sqrt{5} + 3\sqrt{5}$ $-\sqrt{3} + \sqrt{5}$ <p>Organize like terms: $5\sqrt{3} - 6\sqrt{3} = -\sqrt{3}$ $-2\sqrt{5} + 3\sqrt{5} = \sqrt{5}$</p>	<p>21. $2\sqrt{2} + 4\sqrt{7} - 7\sqrt{2} + \sqrt{7}$</p>	<p>22. $8\sqrt{11} + 2\sqrt{10} - 4\sqrt{11} + 6\sqrt{10}$</p>

<p>Estimate. Round to the nearest tenth.</p> <p>$\sqrt{14}$</p> <p>A. 3.6 B. 3.7 C. 3.8</p> <table style="margin-left: 20px;"> <tr> <td style="text-align: center;">3.6</td> <td style="text-align: center;">3.7</td> <td style="text-align: center;">3.8</td> </tr> <tr> <td style="text-align: center;"><u>x3.6</u></td> <td style="text-align: center;"><u>x3.7</u></td> <td style="text-align: center;"><u>x3.8</u></td> </tr> <tr> <td style="text-align: center;">216</td> <td style="text-align: center;">259</td> <td style="text-align: center;">304</td> </tr> <tr> <td style="text-align: center;"><u>+1080</u></td> <td style="text-align: center;"><u>+1110</u></td> <td style="text-align: center;"><u>+1140</u></td> </tr> <tr> <td style="text-align: center;">12.96</td> <td style="text-align: center;">13.69</td> <td style="text-align: center;">14.44</td> </tr> <tr> <td style="text-align: center;">^</td> <td style="text-align: center;">^</td> <td style="text-align: center;">^</td> </tr> </table> <p>too far 14-13.69 14.44-14 =0.31 away =.44 away</p> <p>The closest answer is: 3.7</p> <p style="text-align: right;"><i>Guess & check. Square each option to see which is closest to what you want: 14.</i></p>	3.6	3.7	3.8	<u>x3.6</u>	<u>x3.7</u>	<u>x3.8</u>	216	259	304	<u>+1080</u>	<u>+1110</u>	<u>+1140</u>	12.96	13.69	14.44	^	^	^	<p>23. $\sqrt{22}$</p> <p>A. 4.5 B. 4.6 C. 4.7</p>	<p>24. $\sqrt{12}$</p> <p>A. 3.4 B. 3.5 C. 3.6</p>
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