

Solving Systems by Substitution

If you don't want to graph the lines, there are two other ways to solve a system: Substitution and Elimination.

Substitution is when you plug one equation into the other in order to replace a variable. Then you solve for the remaining variable. Then, plug it in to get the sad replaced variable's value.

First, we'll work with SUBSTITUTION. But, beware: there won't always be an answer and sometimes, the answer will be infinitely many solutions.

<p>EXAMPLE: Solve using substitution.</p> $\begin{cases} 2x - y = 3 \\ x = 3y - 16 \end{cases}$ <p>Plug $x = 3y - 16$ into $2x - y = 3$. Then solve for y.</p> $\begin{aligned} 2(3y - 16) - y &= 3 \\ 6y - 32 - y &= 3 \\ 5y - 32 &= 3 \\ 5y &= 35 \\ y &= 7 \end{aligned}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">The solution is (5, 7)</div> <p>Now, plug y in to find x!</p> $x = 3(7) - 16 = 21 - 16 = 5$	<p>EXAMPLE: Solve using substitution.</p> $\begin{cases} y = 2x + 5 \\ -4x + 2y = 8 \end{cases}$ <p>Plug $y = 2x + 5$ into $-4x + 2y = 8$. Then solve for x.</p> $\begin{aligned} -4x + 2(2x + 5) &= 8 \\ -4x + 4x + 10 &= 8 \\ 10 &= 8 \end{aligned}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">NO SOLUTION!</div> <p>Wait...WHAT?!?! That's never true!</p> <p>So...INCONSISTENT!</p>
<p>1. Solve using substitution.</p> $\begin{cases} 5x + 4y = -7 \\ y = 2x + 8 \end{cases}$	<p>2. Solve using substitution.</p> $\begin{cases} x = 1 - y \\ 3x + 4y = 4 \end{cases}$
<p>3. Solve using substitution.</p> $\begin{cases} 2y + 8x = 4 \\ y = -4x + 2 \end{cases}$	<p>4. Solve using substitution.</p> $\begin{cases} x = 3y - 3 \\ x + y = 9 \end{cases}$
<p>5. Solve using substitution.</p> $\begin{cases} 7x + 2y = -6 \\ x = y - 6 \end{cases}$	<p>6. Solve using substitution.</p> $\begin{cases} y = 6x + 2 \\ 12x - 2y = -2 \end{cases}$

For each of the examples on the last page, one of the equations was already solved for a variable. However, it isn't always that simple. Often, you have to solve for a variable *before* you can use it.

<p>EXAMPLE: Solve using substitution.</p> $\begin{cases} 3x + y = 5 \\ -4x + 6y = 8 \end{cases}$ <p>The easiest variable to solve for is <u>y</u> in the <u>1st</u> equation.</p> $3x + y = 5$ $y = -3x + 5$ <p>Now, plug that in for y in the other equation.</p> $-4x + 6y = 8$ $-4x + 6(-3x + 5) = 8$ $-4x - 18x + 30 = 8$ $-22x + 30 = 8$ $-22x = -22$ $x = 1$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">The solution is (1, 2)</div> <p style="text-align: center;"><i>plug in x = 1 to get y.</i></p> $3(1) + y = 5$ $3 + y = 5$ $y = 2$	<p>EXAMPLE: Solve using substitution.</p> $\begin{cases} 8y = 4x + 16 \\ 3x = 6y - 12 \end{cases}$ <p>The easiest variable to solve for is <u>x</u> in the <u>2nd</u> equation.</p> $3x = 6y - 12$ $x = 2y - 4$ <p>Now, plug that in for x in the other equation.</p> $8y = 4x + 16$ $8y = 4(2y - 4) + 16$ $8y = 8y - 16 + 16$ $8y = 8y$ $y = y$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto; text-align: center;">WAIT!!! y = y is ALWAYS TRUE!</div> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto; text-align: center;">There are INFINITELY MANY solutions!</div>
<p>7. Solve using substitution.</p> $\begin{cases} x - 4 = 3y \\ y + 2x = 22 \end{cases}$ <p>The easiest variable to solve for is ___ in the ___ equation.</p>	<p>8. Solve using substitution.</p> $\begin{cases} 3x + 3y = 6 \\ 2y = -4x + 2 \end{cases}$ <p>The easiest variable to solve for is ___ in the ___ equation.</p>
<p>9. Solve using substitution.</p> $\begin{cases} 3y - 2x = 9 \\ 2x + 4y = 26 \end{cases}$ <p>The easiest variable to solve for is ___ in the ___ equation.</p>	<p>10. Solve using substitution.</p> $\begin{cases} y - 4 = -2(x + 3) \\ y - 5 = -2x - 1 \end{cases}$ <p>The easiest variable to solve for is ___ in the ___ equation.</p>

Now, let's practice testing to see if a given point is a solution to the system. Plug the point into **all of the equations**. It has to work for **all of them** to be a solution.

<p>11. Is (4, -1) a solution to the system?</p> $\begin{cases} y + 7 = 3(x - 2) \\ y = -2x + 4 \end{cases}$	<p>12. Is (1, 0) a solution to the system?</p> $\begin{cases} 6x + 2y = 6 \\ y - 6 = -3(x + 1) \end{cases}$	<p>13. Is (-2, 3) a solution to the system?</p> $\begin{cases} y = 4x + 2 \\ y = -2x - 1 \end{cases}$
<p>14. Is (9, 5) a solution to the system?</p> $\begin{cases} x - y = 4 \\ 2x + 5y = 43 \end{cases}$	<p>15. Is (6, 1, 0) a solution to the system?</p> $\begin{cases} x + 2y + z = 8 \\ 2x - 3y + 4z = 7 \\ -2x + 2y + 2z = -10 \end{cases}$	<p>16. Is (-2, 2, -2) a solution to the system?</p> $\begin{cases} x - y - z = -2 \\ x + y + z = -2 \\ 2x + 3y + z = 0 \end{cases}$