Name: ______________________________ Per: ____

Introduction to Imaginary Numbers

Today, we're going to work with square roots. All the old rules will apply…except one. As of today, you're allowed to square root a negative number.

EXAMPLE $(\sqrt{2})^2 = \sqrt{2}\sqrt{2} = \boxed{2}$	1. $(\sqrt{3})^2 = ?$	2. $(\sqrt{4})^2 = ?$	3. $(\sqrt{5})^2 = ?$	4. $(\sqrt{6})^2 = ?$
EXAMPLE $(\sqrt{-2})^2 = \sqrt{-2}\sqrt{-2}$ $= \boxed{-2}$	5. $(\sqrt{-3})^2 = ?$	6. $(\sqrt{-4})^2 = ?$	7. $(\sqrt{-5})^2 = ?$	8. $(\sqrt{-6})^2 = ?$
EXAMPLE $(\sqrt{2})^3 = \sqrt{2}\sqrt{2}\sqrt{2}$ $= \boxed{2\sqrt{2}}$	9. $(\sqrt{3})^3 = ?$	10. $(\sqrt{5})^3 = ?$	11. $(\sqrt{6})^3 = ?$	12. $(\sqrt{7})^3 = ?$
EXAMPLE $(\sqrt{-2})^3$ $= \sqrt{-2}\sqrt{-2}\sqrt{-2}$ $= \boxed{-2\sqrt{-2}}$	13. $(\sqrt{-3})^3 = ?$	14. $(\sqrt{-5})^3 = ?$	15. $(\sqrt{-6})^3 = ?$	16. $(\sqrt{-7})^3 = ?$

EXAMPLE $(\sqrt{-1})^1$ $= \boxed{\sqrt{-1}}$	17. $(\sqrt{-1})^2$ $= \sqrt{-1}\sqrt{-1}$ $= ?$	18. $(\sqrt{-1})^3 = ?$	19. $(\sqrt{-1})^4 = ?$	20. $(\sqrt{-1})^5 = ?$	21. $(\sqrt{-1})^6 = ?$	22. $(\sqrt{-1})^7 = ?$	23. $(\sqrt{-1})^8 = ?$

Technically, $\sqrt{-1}$ is not physically possible. There are no REAL numbers that could possibly use $\sqrt{-1}$.
This is why $\sqrt{-1}$ is called an IMAGINARY number. And, whenever there's a square root of a negative, you have to replace $\sqrt{-1}$ with i.

$\sqrt{-1} = i$ $\quad (\sqrt{-1})^2 = i^2$ $\quad (\sqrt{-1})^3 = i^3$ …and so on.

Here are examples to show you what simplifying *i* looks like.

i^1 $= \sqrt{-1}$ $= \boxed{i}$	i^2 $= (\sqrt{-1})^2$ $= \boxed{-1}$	i^3 $= (\sqrt{-1})^3$ $= (\sqrt{-1})^2\sqrt{-1}$ $= (-1)\sqrt{-1}$ $= -\sqrt{-1}$ $= \boxed{-i}$	i^4 $= (\sqrt{-1})^4$ $= (\sqrt{-1})^2(\sqrt{-1})^2$ $= (-1)(-1)$ $= \boxed{1}$
i^5 $= i^4 i$ $= (1)\sqrt{-1}$ $= \boxed{i}$	i^6 $= i^4 i^2$ $= (1)(\sqrt{-1})^2$ $= \boxed{-1}$	i^7 $= i^4 i^3$ $= (1)(\sqrt{-1})^3$ $= (\sqrt{-1})^2\sqrt{-1}$ $= (-1)\sqrt{-1}$ $= -\sqrt{-1}$ $= \boxed{-i}$	i^8 $= i^4 i^4$ $= (i^4)^2$ $= (1)^2$ $= \boxed{1}$

EXAMPLE $i^9 = ?$ $= i^8 i = (i^4)^2 i$ $= (1)^2\sqrt{-1}$ $= \boxed{i}$	24. $i^{10} = ?$	**EXAMPLE** $i^{11} = ?$ $= i^8 i^3 = (i^4)^2 i^3$ $= (1)^2\left(\sqrt{-1}\right)^3$ $= \left(\sqrt{-1}\right)^2\sqrt{-1}$ $= (-1)\sqrt{-1}$ $= \boxed{-i}$	25. $i^{12} = ?$
26. $i^{13} = ?$	**EXAMPLE** $i^{14} = ?$ $= i^{12} i^2 = (i^4)^3 i^2$ $= (1)^3\left(\sqrt{-1}\right)^2$ $= \boxed{\mathbf{-1}}$	27. $i^{15} = ?$	**EXAMPLE** $i^{16} = ?$ $= (i^4)^4$ $= (1)^4$ $= \boxed{1}$

So far, they've all been in order. Now, we're going to mix things up a bit. Don't worry, though—I'll give you a trick to make things easier.

If you multiply by 1 or 1^2 or 1^{307}, the problem doesn't change at all.
$i^4 = 1$…so i^4 doesn't change the problem at all either.

THIS MEANS…

You can always get rid of i^4 —whatever REMAINS is the problem you're actually solving. The i^4 is just extra.

So, here's **the trick**:

Divide the exponent by 4.
The REMAINDER is the only part of the exponent you need. The rest can just be canceled out.

EXAMPLE $i^{282} = ?$ $282 \div 4 = 70$ R2 $4 \overline{)282}$ -28 02 - 0 2 Remainder $i^{282} = i^{280} i^2$ $i^{282} = (i^4)^{70} i^2$ $i^{282} = (1)^{70}\left(\sqrt{-1}\right)^2$ $i^{282} = \boxed{-1}$	**Same problem…the short version:**		
	EXAMPLE $i^{282} = ?$	70 R2 $4 \overline{)282}$ -28 02 - 0 2 Remainder	*Ignore everything except the remainder…* $i^{282} = i^2 = \boxed{-1}$

EXAMPLE $i^{51} = ?$ 12 R3 $4 \overline{)51}$ -4 11 - 8 3 Remainder $i^{51} = i^3$ $i^3 = \sqrt{-1}\sqrt{-1}\sqrt{-1}$ $i^3 = (-1)\sqrt{-1}$ $i^3 = \boxed{-i}$	28. $i^{47} = ?$	29. $i^{26} = ?$	30. $i^{111} = ?$
EXAMPLE $i^{576} = ?$ 144 $4 \overline{)576}$ -4 17 -16 16 -16 0 Remainder $i^{576} = i^0$ $i^0 = \boxed{1}$	31. $i^{205} = ?$	32. $i^{342} = ?$	33. $i^{19} = ?$