Name: _____ Per: ____

Simplifying Complex Numbers

Complex Numbers are binomials (expressions with two terms) that look like: $a + bi$. They add and subtract just like any other variable expression—by combining like terms. But, remember: unlike *x*, the *i* term goes at the back.

EXAMPLE $(4 + 3i) - (2 - 4i)$ $4 + 3i - 2 + 4i$ *Re-organized:* $4 - 2 + 3i + 4i$ $\boxed{2 + 7i}$	1. $(9 - 2i) + (4 - i)$	2. $(2 + 4i) - (7 + 5i)$	3. $(3 - 8i) - (6 - 7i)$	4. $(7 + i) + (2 - 3i)$
EXAMPLE $-(5 - 2i) + (4 + 6i)$ $-5 + 2i + 4 + 6i$ *Re-organized:* $-5 + 4 + 2i + 6i$ $\boxed{-1 + 8i}$	5. $-(6 + 4i) + (1 - i)$	6. $(17 + 8i) + (2 + 9i)$	7. $(11 + 3i) - (5 - 3i)$	8. $-(8 + 7i) - (2 - 5i)$

To multiply complex numbers, you have two choices (just like with any variable expressions): you can FOIL or use the box method. I prefer the box method, because it keeps me organized. Either way, you multiply it just like you normally would. However, there are two things you cannot forget: if you can simplify, you must simplify; and $\boldsymbol{i^2 = -1}$.

EXAMPLE $(-2 + 3i)(4 - 5i)$ -2 $+3i$ **4** | -8 | $+12i$ $-5i$ | $+10i$ | $-15i^2$ $-8 + 12i + 10i - 15i^2$ $-8 + 22i - 15(-1)$ $-8 + 22i + 15$ $\boxed{7 + 22i}$	9. $(8 + 2i)(5 - 3i)$	10. $(-4 - i)(7 - 2i)$	11. $(3 + 6i)(-3 + 5i)$
EXAMPLE $(8 + 5i)(6 + 2i)$ **8** $+5i$ **6** | 48 | $+30i$ $+2i$ | $+16i$ | $+10i^2$ $48 + 30i + 16i + 10i^2$ $48 + 46i + 10(-1)$ $48 + 46i - 10$ $\boxed{38 + 46i}$	12. $(11 - 2i)(6 + 3i)$	13. $(-9 + 2i)(9 + 2i)$	14. $(5 - 4i)(2 + 4i)$
EXAMPLE $(-2 - 7i)(-2 + 7i)$ -2 $-7i$ -2 | 4 | $+14i$ $+7i$ | $-14i$ | $-49i^2$ $4 + 14i - 14i - 49i^2$ $4 - 49(-1)$ $4 + 49$ $\boxed{53}$	15. $(4 + 6i)(4 - 6i)$	16. $(-3 - 5i)(-3 + 5i)$	17. $(8 + 2i)(8 - 2i)$

Look at numbers 15-17. What happens when you multiply a complex number with its conjugate (matching complex numbers with the middle sign switched: $a + bi$ & $a - bi$)? _____

To simplify a fraction with a complex number in the numerator (the top), simply reduce each term, as always.

EXAMPLE

$$\frac{9-6i}{12}$$

EVERY term can be reduced by 3, so…

$$\frac{9^{\div 3}-6i^{\div 3}}{12^{\div 3}}=\boxed{\frac{3-2i}{4}}$$

18.

$$\frac{-2+8i}{-10}$$

19.

$$\frac{14+21i}{14}$$

20.

$$\frac{5+5i}{5}$$

21.

$$\frac{-12+16i}{20}$$

However, you **cannot have a complex number in the denominator (the bottom)**. To get rid of it, multiply the top and the bottom by the *complex conjugate* of the bottom (the same complex number with the middle sign switched).

EXAMPLE Simplify.

$$\frac{3}{2-5i}$$

conjugate:

$2-5i \rightarrow 2+5i$

$$\frac{3}{2-5i}\cdot\frac{2+5i}{2+5i}$$

$$\frac{3(2+5i)}{(2-5i)(2+5i)}$$

	2	$\mathbf{-5i}$
2	4	$-10i$
$\mathbf{+5i}$	$+10i$	$-25i^2$

$4-10i+10i-25i^2$

$=4-25(-1)$

$=4+25$

$=29$

$$=\boxed{\frac{6+15i}{29}}$$

22. Simplify.

$$\frac{5}{7+2i}$$

23. Simplify.

$$\frac{-2}{4-6i}$$

EXAMPLE Simplify.

$$\frac{8}{-3+6i}$$

conjugate:

$-3+6i \rightarrow -3-6i$

$$\frac{8}{-3+6i}\cdot\frac{-3-6i}{-3-6i}$$

$$\frac{8(-3-6i)}{(-3+6i)(-3-6i)}$$

	$\mathbf{-3}$	$\mathbf{+6i}$
$\mathbf{-3}$	9	$-18i$
$\mathbf{-6i}$	$+18i$	$-36i^2$

$9-18i+18i-36i^2$

$=9-36(-1)$

$=9+36$

$=45$

$$\frac{-24-48i}{45}$$

Reduce by 3!

$$\frac{-24^{\div 3}-48i^{\div 3}}{45^{\div 3}}=\boxed{\frac{-8+16i}{15}}$$

24. Simplify.

$$\frac{9}{2+8i}$$

25. Simplify.

$$\frac{-5}{12-6i}$$

EXAMPLE Simplify.

$$\frac{5i}{7-i}$$

conjugate:

$7-i \rightarrow 7+i$

$$\frac{5i}{7-i}\cdot\frac{7+i}{7+i}$$

$$\frac{5i(7+i)}{(7-i)(7+i)}$$

	7	$\mathbf{-i}$
7	49	$-7i$
$\mathbf{+i}$	$+7i$	$-i^2$

$49-7i+7i-i^2$

$=49-(-1)$

$=49+1$

$=50$

$$\frac{35i+5i^2}{50}$$

Simplify the top!

$$\frac{35i+5(-1)}{50}=\frac{35i-5}{50}=\frac{-5+35i}{50}$$

Reduce by 5!

$$\frac{-5^{\div 5}+35i^{\div 5}}{50^{\div 5}}=\boxed{\frac{-1+7i}{10}}$$

26. Simplify.

$$\frac{2i}{5-3i}$$

27. Simplify.

$$\frac{-6i}{2+4i}$$