

Factoring to Determine Solutions

You know how to **factor (when a = 1)**—find two numbers that **multiply to equal c**, and **add to equal b**.

The next step is to use factoring to solve for x. This is only possible under two conditions:

1. The quadratic must be set equal to 0
2. The quadratic must be factorable
—sometimes, there is no set of numbers that multiplies to c and adds to b. It's sad, but true.
If this is the case, you cannot factor it, so you cannot solve for x this way.

There is one other thing that you must remember:

If two terms **multiply to 0**, then **one of 'em must be zero**.

Example: $(5)(?) = 0$...umm, the ? has to be 0. That's the only way to get $\boxed{0}$.

Another example: $(5)(x) = 0$...umm, that's the same problem. x is still $\boxed{0}$.

Another example: $(5)(x + 1) = 0$...oooh, tricky. I still have to multiply 5 by 0.

This means that $x + 1 = 0$. So, $x = \boxed{-1}$

Yet another example: $(x - 5)(x + 1) = 0$...you can't fool me. We've done this before. One of them has to be 0.

So, this means that either $x - 5 = 0$ or $x + 1 = 0$

$x = 5$ or $x = -1$

$x = \boxed{\{-1, 5\}}$ I win.

<p>EXAMPLE Solve for x. $0 = x^2 - 16x + 48$ <i>Two numbers that multiply to $c = 48$ and add to $b = -16$...</i> <i>Guess and check:</i> $(-2)(-24) = 48$ $-2 + -24 = -16$...umm, no. <i>NEXT!</i> $(-4)(-12) = 48$ $-4 + -12 = -16$ <i>YAHTZEE!</i> <i>Use: -4 & -12</i> $0 = x^2 - 16x + 48$ $0 = (x - 4)(x - 12)$ $x - 4 = 0$ or $x - 12 = 0$ $x = 4$ or $x = 12$ $x = \boxed{\{4, 12\}}$</p>	<p>EXAMPLE Solve for x. $0 = x^2 + 9x - 14$ <i>Two numbers that multiply to $c = -14$ and add to $b = +9$...</i> <i>Guess and check:</i> $(-2)(7) = -14$ $-2 + 7 = 9$...nope. <i>NEXT!</i> $(2)(-7) = -14$ $2 + -7 = 9$...still no. <i>NEXT!</i> $(-1)(14) = -14$ $-1 + 14 = 9$...no. <i>NEXT!</i> $(1)(-14) = -14$ $1 + -14 = 9$...no. <i>NEXT!</i> <i>Okay, there are no other numbers that multiply to -14. I'm out of options.</i> <i>This problem is <u>NOT POSSIBLE</u>.</i></p>	<p>EXAMPLE Solve for x. $0 = x^2 - x - 12$ <i>Two numbers that multiply to $c = -12$ and add to $b = -1$...</i> <i>Guess and check:</i> $(-3)(4) = -12$ $-3 + 4 = -1$...nope. <i>NEXT!</i> $(3)(-4) = -12$ $3 + -4 = -1$ <i>Found 'em!</i> <i>Use: 3 & -4</i> $0 = x^2 - x - 12$ $0 = (x + 3)(x - 4)$ $x + 3 = 0$ or $x - 4 = 0$ $x = -3$ or $x = 4$ $x = \boxed{\{-3, 4\}}$</p>
<p>1. Solve for x. $0 = x^2 + 9x - 36$</p>	<p>2. Solve for x. $0 = x^2 - 7x + 10$</p>	<p>3. Solve for x. $0 = x^2 + 2x + 24$</p>
<p>4. Solve for x. $0 = x^2 - 5x - 6$</p>	<p>5. Solve for x. $0 = x^2 + 8x - 15$</p>	<p>6. Solve for x. $0 = x^2 - 2x - 48$</p>

7. Solve for x . $0 = x^2 + x - 30$	8. Solve for x . $0 = x^2 - 3x - 54$	9. Solve for x . $0 = x^2 + 12x - 20$
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Now, we'll practice a skill you'll need soon. I've already shown it to you: *the magic step*. This time, I want you to do the magic step and then solve for x . Here's how you'll do it:

1. Use the Magic Step
2. Simplify the right side
3. Square root both sides (don't forget the \pm)
4. Get x alone.

<p>EXAMPLE</p> $x^2 + 12x = 13$ <p>MAGIC $\rightarrow 12 \div 2 = 6, so ...$</p> $(x + 6)^2 = 13 + (6)^2$ $(x + 6)^2 = 13 + 36$ $(x + 6)^2 = 49$ $\sqrt{(x + 6)^2} = \pm\sqrt{49}$ $x + 6 = \pm 7$ $x = -6 \pm 7$ <p>So, $x = -6 - 7 = -13$ or $x = -6 + 7 = 1$</p> $x = \boxed{\{-13, 1\}}$	<p>EXAMPLE</p> $x^2 - 2x = -5$ <p>MAGIC $\rightarrow -2 \div 2 = -1, so ...$</p> $(x - 1)^2 = -5 + (-1)^2$ $(x - 1)^2 = -5 + 1$ $(x - 1)^2 = -4$ $\sqrt{(x - 1)^2} = \pm\sqrt{-4}$ $x - 1 = \pm 2i$ $x = \boxed{1 \pm 2i}$	<p>EXAMPLE</p> $x^2 + 8x = -23$ <p>MAGIC $\rightarrow 8 \div 2 = 4, so ...$</p> $(x + 4)^2 = -23 + (4)^2$ $(x + 4)^2 = -23 + 16$ $(x + 4)^2 = -7$ $\sqrt{(x + 4)^2} = \pm\sqrt{-7}$ $x + 4 = \pm i\sqrt{7}$ $x = \boxed{-4 \pm i\sqrt{7}}$
10. $x^2 + 20x = -19$	11. $x^2 - 14x = 23$	12. $x^2 - 4x = -20$
13. $x^2 - 18x = -12$	14. $x^2 + 6x = -9$	15. $x^2 + 16x = -25$