

Factoring...when $a \neq 1$

You know how to factor problems like $f(x) = x^2 - 3x + 2$. All you have to do is find two numbers that multiply to $c = +2$ and add to $b = -3 \rightarrow -1$ & -2 . So, your factors would be $f(x) = (x - 1)(x - 2)$.

The process for factoring problems when $a \neq 1$ is very similar...with an added step or two.

Step 1: Find two numbers that multiply to ac and add to b .

Step 2: Split bx into two pieces using these two numbers.

EXAMPLE: Factor $f(x) = 2x^2 + 2x - 4$

$ac = (2)(-4) = -8$	$b = 2$
Factors of -24:	Do they add to +2?
1 & -8 -1 & 8	1 + -8 NO -1 + 8 NO
2 & -4 -2 & 4	2 + -4 NO -2 + 4 YES
<i>I found it, so I can stop!</i>	

Use these to split up the bx term

$f(x) = 2x^2 + 2x - 4$
$f(x) = 2x^2 - 2x + 4x - 4$

Step 3: Factor out what you can from the group of the first two terms & from the group of the second two terms

The goal is to make what's left in each group the same!

EXAMPLE (cont'd): $f(x) = 2x^2 - 2x + 4x - 4$

Group 1:	$2x^2 - 2x$	Group 2:	$4x - 4$
They're both divisible by $2x$ — factor it out!	$2x(x - 1)$	They're both divisible by 4 — factor it out!	$4(x - 1)$

Notice that when I factor them, I end up with $(x - 1)$ for both groups!

$$f(x) = 2x(x - 1) + 4(x - 1)$$

Step 4: Do the opposite of distribution, and factor the matching group out.

$$\boxed{f(x) = (x - 1)(2x + 4)}$$

<p>EXAMPLE Factor. $f(x) = 3x^2 + 7x + 2$</p> <p>$ac = (3)(2) = 6$ $b = 7$ Factors of 6: 1 & 6 YES!!!</p> <p>$f(x) = 3x^2 + 1x + 6x + 2$ $f(x) = 3x^2 + x$ + $6x + 2$ $f(x) = x(3x + 1)$ + $2(3x + 1)$ $f(x) = (3x + 1)(x + 2)$</p>	<p>1. Factor. $f(x) = 5x^2 - 8x - 4$</p>	<p>2. Factor. $f(x) = 2x^2 + 8x + 6$</p>
<p>EXAMPLE Factor. $f(x) = 15x^2 + 7x - 2$</p> <p>$ac = (15)(-2) = -30$ $b = 7$ Factors of -30: 1 & -30 NO -1 & 30 NO 2 & -15 NO -2 & 15 NO 3 & -10 NO -3 & 10 YES!!!</p> <p>$f(x) = 15x^2 - 3x + 10x - 2$ $f(x) = 15x^2 - 3x$ + $10x - 2$ $f(x) = 3x(5x - 1)$ + $2(5x - 1)$ $f(x) = (5x - 1)(3x + 2)$</p>	<p>3. $f(x) = 2x^2 + 2x - 12$</p>	<p>4. $f(x) = 14x^2 + 25x + 3$</p>

Now, we'll add the next step—solving quadratics. Factor and solve for the zeros.

<p>EXAMPLE $0 = 4x^2 - 16x + 7$</p> <p>$ac = (4)(7) = 28 \quad b = -16$ Factors of 28: -1 & -28 NO -2 & -14 YES!!!</p> <p>$0 = 4x^2 - 2x - 14x + 7$ $0 = 4x^2 - 2x - 14x + 7$</p> <p><i>We could factor +7 from the end...</i> $0 = 2x(2x - 1) + 7(-2x + 1)$</p> <p><i>BUT we want the (group) to match...</i> <i>So, factor out -7 instead!</i> $0 = 2x(2x - 1) - 7(2x - 1)$ $0 = (2x - 1)(2x - 7)$</p> <p>Now, split it up and solve. $2x - 1 = 0 \quad \text{or} \quad 2x - 7 = 0$ $2x = 1 \quad \quad \quad 2x = 7$ $x = \frac{1}{2} \quad \quad \quad x = \frac{7}{2}$</p> <p>$x = \left\{ \frac{1}{2}, \frac{7}{2} \right\}$</p>	<p>5. $0 = 8x^2 - 37x - 15$</p>	<p>6. $0 = 3x^2 + 19x + 28$</p>
<p>EXAMPLE $0 = 10x^2 + 11x - 18$</p> <p>$ac = (10)(-18) = -180 \quad b = 8$ Factors of 28: -1 & 180 NO 1 & -180 NO -9 & 20 YES!!!</p> <p>$0 = 10x^2 - 9x + 20x - 18$ $0 = 10x^2 - 9x + 20x - 18$ $0 = x(10x - 9) + 2(10x - 9)$ $0 = (10x - 9)(x + 2)$</p> <p>Now, split it up and solve. $10x - 9 = 0 \quad \text{or} \quad x + 2 = 0$ $10x = 9 \quad \quad \quad x = -2$ $x = \frac{9}{10}$</p> <p>$x = \left\{ -2, \frac{9}{10} \right\}$</p>	<p>7. $0 = 4x^2 + 42x - 22$</p>	<p>8. $0 = 6x^2 + 13x + 6$</p>