

Operations on Polynomials

Addition & Subtraction

To add or subtract polynomials, all you have to do is to *combine like terms*. This means that you have to add or subtract the terms that have matching variables (x^9 matches x^9 **not** x^8).

Remember: when you subtract a group, everything in the parentheses is being subtracted.

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| <p>EXAMPLE $(2n^7 - 4n^5 + 6n^2 + 1) + (2n^5 - 7)$ <i>If you re-order them by degree (or total exponential power), then the like terms will line up. Now, simplify!</i> $2n^7 - 4n^5 + \underline{2n^5} + 6n^2 + 1 + \underline{-7}$ $\boxed{2n^7 - 2n^5 + 6n^2 - 6}$</p> | <p>EXAMPLE $(9x^3 + 4x^2 - 9) - (6x^4 - 2x^3 + 5)$ <i>Re-write it so that the negative belongs to every term in the group.</i> $9x^3 + 4x^2 - 9 - (6x^4) - (-2x^3) - (+5)$ $9x^3 + 4x^2 - 9 - 6x^4 + 2x^3 - 5$ <i>Re-order them by degree, & simplify!</i> $\underline{-6x^4} + 9x^3 + \underline{2x^3} + 4x^2 - 9 - \underline{5}$ $\boxed{-6x^4 + 11x^3 + 4x^2 - 14}$</p> | <p>EXAMPLE $-(4a^3 + 2a) - (-5a^2 + 8a)$ $-(4a^3) - (+2a) - (-5a^2) - (+8a)$ $-4a^3 - 2a + 5a^2 - 8a$ $-4a^3 + 5a^2 - 2a - 8a$ $\boxed{-4a^3 + 5a^2 - 10a}$</p> |
| <p>1. $(9x^6 + 12x^2 - 3) + (5x^6 + 8)$</p> | <p>2. $(5p^7 - 3p^5 - 4) - (4p^7 - 2p^5)$</p> | <p>3. $(11b^9 - 3b + 7) + (5b^7 - 8)$</p> |
| <p>4. $(10z^2 + 3z) - (-8z^3 - 2z^2 + 2)$</p> | <p>5. $-(8x^5 + 7x^2) + (6x^2 + 15)$</p> | <p>6. $-(-3h - 6) - (2h^2 + 7h)$</p> |
| <p>7. $(7k^{12} + 2k^2) - (7k^{12} + 2k^2 - 6k)$</p> | <p>8. $(r^9 - r^4 + r^2 + 3) + (r^5 - 2r^2)$</p> | <p>9. $-(7x^5 + 8x^3) - (3x^3 - 6)$</p> |

Multiplication

The easiest way to multiply polynomials is to use the box method. Multiply the terms, then add 'em up.

Remember: when you multiply matching bases, the exponents *add together*.

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| <p>EXAMPLE $-5a^3b(-4a^2b + 2ab - 3ab^2)$ $-5a^3b \begin{array}{ c c c } \hline -4a^2b & +2ab & -3ab^2 \\ \hline \end{array}$ $\begin{array}{ c c c } \hline 20a^5b^2 & -10a^4b^2 & +15a^4b^3 \\ \hline \end{array}$ <i>Now, just add 'em up!</i> $\boxed{20a^5b^2 - 10a^4b^2 + 15a^4b^3}$</p> | <p>EXAMPLE $(2x^3 + 4x)(7x^2 + 3x - 9)$ $\begin{array}{ c c c } \hline 7x^2 & +3x & -9 \\ \hline \end{array}$ $2x^3 \begin{array}{ c c c } \hline 14x^5 & +6x^4 & -18x^3 \\ \hline \end{array}$ $+4x \begin{array}{ c c c } \hline +28x^3 & +12x^2 & -36x \\ \hline \end{array}$ $14x^5 + 6x^4 - 18x^3 + 28x^3 + 12x^2 - 36x$ $\boxed{14x^5 + 6x^4 + 10x^3 + 12x^2 - 36x}$</p> | <p>EXAMPLE $(5q - 3)(6q^2 - 5q + 7)$ $\begin{array}{ c c c } \hline 6q^2 & -5q & +7 \\ \hline \end{array}$ $5q \begin{array}{ c c c } \hline 30q^3 & -25q^2 & +35q \\ \hline \end{array}$ $-3 \begin{array}{ c c c } \hline -18q^2 & +15q & -21 \\ \hline \end{array}$ $30q^3 - 25q^2 + 35q - 18q^2 + 15q - 21$ $30q^3 - 25q^2 - 18q^2 + 35q + 15q - 21$ $\boxed{30q^3 - 43q^2 + 50q - 21}$</p> |
| <p>10. $(5r^4 + 8)(9r^3 + 12r^2 - 3)$</p> | <p>11. $(4m^3 - 2m^2)(5m^4 - 3m^3 - 4)$</p> | <p>12. $3x^5(11x^9 - 3x + 7)$</p> |

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| 13. $(10p^2 + 3p)(-8p^3 - 2p^2 + 2)$ | 14. $-7gh^3(2g^3h + 4g^2h^2 - 9gh^3)$ | 15. $(-3x - 6)(2x^2 + 7x)$ |
| 16. $-3d^2e^2(7d^5e^3 - 8de^4)$ | 17. $(x^9 - x^4 + x^2 + 3)(x^5 - 2x^2)$ | 18. $(7x^5 + 8x^3)(3x^3 - 6)$ |

Simplifying Rational Polynomials

First, factor out anything you can from the numerator and the denominator.

Second, cancel out any terms that you can (*must be attached by multiplication **not** addition in order to cancel*).

Finally, if you need to, multiply what's left back together.

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| <p>EXAMPLE</p> $\frac{8x - 6}{4} = \frac{(2)(4x - 3)}{2(2)} = \frac{\boxed{4x - 3}}{2}$ | <p>EXAMPLE</p> $\frac{3x^2 - 9x}{6x^2} = \frac{3x(x - 3)}{3x(2x)} = \frac{\boxed{x - 3}}{2x}$ | <p>EXAMPLE</p> $\frac{3x^2 + 7x + 2}{4x + 8}$ <p>Factor the top. $AC = (3)(2) = 6$ & $B = 7$ Numbers that mult. to AC & add to B...</p> $\frac{3x^2 + 6x + 1x + 2}{4x + 8}$ $= \frac{3x(x + 2) + 1(x + 2)}{4(x + 2)}$ $= \frac{(3x + 1)(x + 2)}{4(x + 2)} = \frac{\boxed{3x + 1}}{4}$ |
| 19. $\frac{18x + 12}{15x + 10}$ | 20. $\frac{8x^4 + 4x^2}{10x^5}$ | 21. $\frac{x^2 + 8x + 12}{3x + 6}$ |
| 22. $\frac{5x^2 + 3x - 8}{3x - 3}$ | 23. $\frac{5x - 15}{-2x^2 + 6x}$ | 24. $\frac{9x^5 + 27x^3}{18x^4}$ |