Name: ______________________________ Per: _____

Binomial Expansion

Binomial Expansion is when you take a two-term expression and multiply it by itself…over and over again. Now, you could simply multiply it…over and over again. However, if you know Pascal's Triangle, there's a shortcut.

Pascal's Triangle

(to find the digits of each row, add the two digits directly above the space you're trying to fill)

0 Power									1								
1^{st} Power								1		1							
2^{nd} Power							1		2		1						
3^{rd} Power						1		3		3		1					
4^{th} Power					1		4		6		4		1				
5^{th} Power				1		5		10		10		5		1			
6^{th} Power			1		6		15		20		15		6		1		
7^{th} Power		1		7		21		35		35		21		7		1	
8^{th} Power	1		8		28		56		70		56		28		8		1

Pascal's triangle gives you the **coefficients** that multiply in front of the two terms. In the example below, the binomial is raised to the **third power**, so we'll use that row. Note how the exponents of the two terms, *x* and *y*, relate to one another and to the original power of the expansion.

EXAMPLE: $(x+y)^3 = 1(x)^3(y)^0 + 3(x)^2(y)^1 + 3(x)^1(y)^2 + 1(x)^0(y)^3$

The first term, *x*, starts with all of the power (3) and slowly loses it, one unit at a time until it has no power at all (0). The second term, *y*, starts with none of the power (0) and slowly gain it, one unit at a time, until it has all of it (3).

Now, we simplify.

EXAMPLE: $(x+y)^3 = 1(x)^3(y)^0 + 3(x)^2(y)^1 + 3(x)^1(y)^2 + 1(x)^0(y)^3$

EXAMPLE: $(x+y)^3 = \boxed{x^3 + 3x^2y + 3xy^2 + y^3}$

EXAMPLE

Expand $(x-2)^4$.

The fourth power row is: 1 4 6 4 1, so the structure will look like…

$$1(\ )^4(\ )^0 + 4(\ )^3(\ )^1 + 6(\ )^2(\ )^2 + 4(\ )^1(\ )^3 + 1(\ )^0(\ )^4$$

Now, let's plug in the two terms…

$$1(x)^4(-2)^0 + 4(x)^3(-2)^1 + 6(x)^2(-2)^2 + 4(x)^1(-2)^3 + 1(x)^0(-2)^4$$

and simplify…

$$x^4 + 4x^3(-2) + 6x^2(4) + 4x(-8) + 16$$

$$\boxed{x^4 - 8x^3 + 24x^2 - 32x + 16}$$

1. Expand $(x+4)^3$.

2. Expand $(x-3)^5$.

3. Expand $(x+1)^6$.

EXAMPLE

Expand $(3x+y)^5$.

The fifth power row is: 1 5 10 10 5 1, so the structure will look like…

$$1(\ )^5(\ )^0+5(\ )^4(\ )^1+10(\ )^3(\ )^2+10(\ )^2(\ )^3+5(\ )^1(\ )^4+1(\ )^0(\ )^5$$

Now, let's plug in the two terms…

$$1(3x)^5(y)^0+5(3x)^4(y)^1+10(3x)^3(y)^2+10(3x)^2(y)^3+5(3x)^1(y)^4+1(3x)^0(y)^5$$

and simplify…

$$(3)^5x^5+5(3)^4x^4y+10(3)^3x^3y^2+10(3)^2x^2y^3+5(3)xy^4+y^5$$

$$243x^5+5(81)x^4y+10(27)x^3y^2+10(9)x^2y^3+15xy^4+y^5$$

$$\boxed{243x^5+405x^4y+270x^3y^2+90x^2y^3+15xy^4+y^5}$$

4. Expand $(2x-y)^3$.

5. Expand $(x+5y)^4$.

6. Expand $(3x-2y)^4$.