Name: ______________________________ Per: ____

Synthetic Substitution
(The Remainder Theorem)

Synthetic Substitution is basically another way to plug numbers into a polynomial and to solve for an answer. To do synthetic substitution, you use the synthetic division puzzle, but instead of putting the zero in the top left box, you write the number you're plugging in. The answer to the problem will be the remainder, which is why this process is also called the **Remainder Theorem**. You do not have to use this method, but it's sometimes easier than plugging in numbers to compute.

For example:
$P(-5)$ for $P(x) = 3x^4 - 7x^3 + 5x^2 - 8x + 10$

The traditional solution is to plug it in:

$$P(-5) = 3(-5)^4 - 7(-5)^3 + 5(-5)^2 - 8(-5) + 10 = 3(625) - 7(-125) + 5(25) + 40 + 10$$

$$P(-5) = 1875 + 875 + 125 + 40 + 10 = 2750 + 165 + 10 = \boxed{2925}$$

Now, let's do the same problem with **SYNTHETIC SUBSTITUTION**:

$P(-5) =$	$3x^4$	$-7x^3$	$+5x^2$	$-8x$	$+10$
↓	↓	↓	↓	↓	↓
-5	3	-7	5	-8	10
	↓	-15	110	-575	2915
	3	-22	115	-583	2925

The answer is in the Remainder box…
$P(5) = \boxed{2925}$

The answer will be the same, no matter which method you choose. Today, you're going to practice **Synthetic Substitution** (The Remainder Theorem).

1. Find $P(2)$ using the Remainder Theorem for $x = 2$.
$P(x) = 3x^4 + 6x^3 - 2x + 5$
Setup:

2	3	6	0	-2	5

2. Find $P(-6)$ using Synthetic Substitution for $x = -6$.
$P(x) = 9x^3 + 36x^2 - 90x - 8$

3. Find $P(4)$ using the Remainder Theorem for $x = 4$.
$P(x) = 7x^5 - 27x^4 + 2x^3 - 20x^2 + 1$

4. Find $P(1)$ using Synthetic Substitution for $x = 1$.
$P(x) = x^5 + 2x^3 - x$

5. Find $P(-3)$ using the Remainder Theorem for $x = -3$.
$P(x) = 7x^4 + 18x^3 - 2x^2$

6. Find $P(8)$ using Synthetic Substitution for $x = 8$.
$P(x) = -8x^4 + 60x^3 + 20x^2 + 100x$

7. Find $P(-5)$ using the Remainder Theorem for $x = -5$. $P(x) = -x^6 + x^5 - x^4 + x^3 - x^2 + 1$	8. Find $P(9)$ using Synthetic Substitution for $x = 9$. $P(x) = 2x^5 - 20x^4 + 24x^3 - 53x^2 + 10x - 5$
9. Find $P(5)$ using the Remainder Theorem for $x = 5$. $P(x) = -5x^3 + 26x^2 - 8x + 9$	10. Find $P(-3)$ using Synthetic Substitution for $x = -3$. $P(x) = x^6 - 20x^4 - 2x^3 + 5x - 8$
11. Find $P(0)$ using the Remainder Theorem for $x = 0$. $P(x) = 6x^2 - 2x + 9$	12. Find $P(-10)$ using Synthetic Substitution for $x = -10$. $P(x) = 3x^3 + 4x^2 - 18$
13. Find $P(6)$ using the Remainder Theorem for $x = 6$. $P(x) = 3x^3 - 16x^2 - 16x + 24$	14. Find $P(5)$ using Synthetic Substitution for $x = 5$. $P(x) = x^3 - 125$