Name: ______________________________ Per.: ______

Factoring Polynomials

There are four ways to factor polynomials. The first method is called **grouping**. This is the method we used to factor quadrilaterals when $a \neq 1$. Basically, you split the polynomial into groups, factor out what you can (hoping to end up with matching factors), then you backwards distribute.

Be careful, though—if there's something that you can factor out first (for example, every term is divisible by 4), then you have to start by factoring it out.

Factor by grouping.

EXAMPLE $x^3 - 2x^2 + 10x - 20$ *First, look at them as two different* ***binomial groups:*** $x^3 - 2x^2 \qquad + 10x - 20$ *Then, factor out what you can from* ***each group.*** $x^2(x-2) \qquad + 10(x-2)$ *Finally, factor out the matching binomial.* $\boxed{(x^2+10)(x-2)}$	1. $6x^3 - 15x^2 + 10x - 25$	2. $6x^3 + 3x^2 + 16x + 8$
EXAMPLE $10x^3 + 25x^2 - 4x - 10$ $10x^3 + 25x^2 \qquad - 4x - 10$ $5x^2(2x+5) \qquad - 2(2x+5)$ $\boxed{(5x^2-2)(2x+5)}$	3. $9x^3 + 9x^2 + 7x + 7$	4. $4x^3 + 6x^2 - 10x - 15$
EXAMPLE $14x^3 + 4x^2 + 7x + 2$ $14x^3 + 4x^2 \qquad + 7x + 2$ *Remember: you have to factor out* ***something****, even if it's 1…* $2x^2(7x+2) \qquad + 1(7x+2)$ $\boxed{(2x^2+1)(7x+2)}$	5. $12x^3 + 18x^2 + 10x + 15$	6. $21x^3 + 35x^2 + 18x + 30$
EXAMPLE $8x^3 + 20x^2 + 6x + 15$ $8x^3 + 20x^2 \qquad + 6x + 15$ $4x^2(2x+5) \qquad + 3(2x+5)$ $\boxed{(4x^2+3)(2x+5)}$	7. $2x^3 + 3x^2 + 8x + 12$	8. $5x^3 - 7x^2 + 25x - 35$

The other three methods for factoring polynomials *only work* on **special binomials**. These methods are called:

Difference of Squares	Sum of Cubes	Difference of Cubes
Factoring when you have a square minus another square… $\boldsymbol{a^2 - b^2}$	*Factoring when you have a cube plus another cube…* $\boldsymbol{a^3 + b^3}$	*Factoring when you have a cube minus another cube…* $\boldsymbol{a^3 - b^3}$

Their factors always follow specific patterns. If you know the patterns, factoring is simple. Without them, though…

Difference of Squares	Sum or Difference of Cubes
(1st root + 2nd root)(1st root – 2nd root)	There are two patterns with these binomials: 1. The terms go in this order: (1st root __ 2nd root)(1st squared ___1st & 2nd ___2nd squared) 2. The signs (plus or minus) go like this: (same sign)(different sign plus sign)
$\boldsymbol{a^2 - b^2 = (a+b)(a-b)}$	$\boldsymbol{a^3 + b^3 = (a+b)(a^2 - ab + b^2)}$ *same* ↑ *diff.*↑ ↑ *plus* $\boldsymbol{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$ *same* ↑ *diff.*↑ ↑ *plus*

Before you can factor special binomials, you have to learn to recognize square and cube roots. You know how to do this with everyday numbers (49 is 7 squared, while 27 is 3 cubed), but most don't completely understand how it applies to variables. Basically, the idea is that to square root an exponent, you divide the exponent by 2. To cube root an exponent, you divide the exponent by three.

Is the term a square, a cube, or neither? If possible, identify the root.

EXAMPLE $144x^8$ $\sqrt{144} = 12,$ $\sqrt{x^8} = x^{8\div 2} = x^4$ $144x^8$ *is a* $\boxed{square}$. *Root:* $\sqrt{144x^8} = \boxed{12x^4}$	**EXAMPLE** $64x^7$ $\sqrt{64} = 8,$ but $\sqrt{x^7} = x^{7\div 2} = \ldots no.$ *It's not a square* $\sqrt[3]{64} = 4,$ but $\sqrt[3]{x^7} = x^{7\div 3} = \ldots no.$ *It's not a cube, either.* *This is* $\boxed{neither}$.	**EXAMPLE** $27x^{12}$ $\sqrt{27} = \ldots no$ *It's not a square.* $\sqrt[3]{27} = 3$ $\sqrt[3]{x^{12}} = x^{12\div 3} = x^4$ $27x^{12}$ *is a* $\boxed{cube}$. *Root:* $\sqrt[3]{27x^{12}} = \boxed{3x^4}$	**EXAMPLE** $169x^{10}$ $\sqrt{169} = 13$ $\sqrt{x^{10}} = x^{10\div 2} = x^5$ $169x^{10}$ *is a* $\boxed{square}$. *Root:* $\sqrt{169x^{10}} = \boxed{13x^5}$
9. $216x^9$	10. $121x^{18}$	11. $125x^{24}$	12. $8x^{15}$
13. $64x^{22}$	14. $225x^8$	15. $343x^{21}$	16. $1000x^{36}$

Identify the type of special binomial (difference of squares, differences of cubes, or sum of cubes).

17. $81x^4 - 25x^2$	18. $27x^9 + 8x^6$	19. $64x^{60} - 125x^6$	20. $x^3 + 1$