Name: _____ Per: _____

Factoring Polynomials Part 2

The three methods for factoring polynomials are:

Difference of Squares	Sum or Difference of Cubes	Grouping
$a^2 - b^2 = (a + b)(a - b)$	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $same \uparrow \quad diff. \uparrow \quad \uparrow plus$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ $same \uparrow \quad diff. \uparrow \quad \uparrow plus$	$ca + cb + da + db$ $= c(a + b) + d(a + b)$ $= (a + b)(c + d)$

The first step to factoring them is to see which of the methods applies to that polynomial. Is it a binomial? If it is, is it a difference (subtraction) or a sum (addition)? Are the terms squares or cubes? If it's not a binomial, your only option is grouping. However, grouping doesn't always work. Sometimes, the polynomial *cannot be factored* using any of these methods.

Once you've identified the type of factoring problem, use the patterns and processes to factor it.

EXAMPLE Factor, if possible. $x^5 + 2x^3 - 3x^2 - 6$ Type: Grouping $x^5 + 2x^3 \quad - 3x^2 - 6$ $x^3(x^2 + 2) \quad - 3(x^2 + 2)$ $(x^3 - 3)(x^2 + 2)$	**EXAMPLE** Factor, if possible. $x^9 - 27$ Type: Difference of Cubes $\sqrt[3]{x^9} = x^3 \quad \sqrt[3]{27} = 3$ *The pattern is:* *(1st__2nd)((1st)²___both___(2nd)²)* $same \uparrow (-) \quad diff. \uparrow (+) \quad \uparrow plus (+)$ $(x^3 - 3)((x^3)^2 + 3x^3 + (3)^2)$ $(x^3 - 3)(x^6 + 3x^3 + 9)$	**EXAMPLE** Factor, if possible. $x^4 - 16$ Type: Difference of Squares $\sqrt{x^4} = x^2 \quad \sqrt{16} = 4$ *The pattern is:* *(1st + 2nd)(1st – 2nd)* $(x^2 + 4)(x^2 - 4)$*....WAIT!!!* $(x^2 + 4)$ *is not factorable, but* $(x^2 - 4)$ *is a difference of squares.* $\sqrt{x} = x$ *&* $\sqrt{4} = 2$ *So...*$(x^2 + 4)(x^2 - 4) =$ $(x^2 + 4)(x + 2)(x - 2)$
1. Factor, if possible. $x^{12} + 8$ Type: _____	2. Factor, if possible. $3x^5 + 6x^4 - 4x - 8$ Type: _____	3. Factor, if possible. $x^6 - 49$ Type: _____
4. Factor, if possible. $5x^3 + 15x^2 + 2x + 6$ Type: _____	5. Factor, if possible. $x^{27} - 1$ Type: _____	6. Factor, if possible. $x^{10} - 1$ Type: _____
7. Factor, if possible. $x^2 + 25$ Type: _____	8. Factor, if possible. $x^{18} + 216$ Type: _____	9. Factor, if possible. $x^{24} + 64$ Type: _____

The next step to solving polynomial problems involving factoring, is being able to factor and cancel rational expressions (fractions involving variables).

First, see if there are any variables or numbers that can be factored out of every term on the top or every term on the bottom. Then, factor what's left. If the top and bottom have any factors in common, you can cancel them out.

EXAMPLE Reduce the fraction as much as possible. $\frac{x^5+2x^3-3x^2-6}{2x^3-6}$ $\frac{x^5+2x^3-3x^2-6 \leftarrow grouping}{2x^3-6 \leftarrow both\ have\ 2s}$ $\frac{x^3(x^2+2)-3(x^2+2)}{2(x^3-3)}$ $\frac{(x^3-3)(x^2+2)}{2(x^3-3)} \leftarrow cancel\ (x^3-3)$ $\boxed{\frac{x^2+2}{2}}$	**EXAMPLE** Reduce the fraction as much as possible. $\frac{x^6+3x^3+9}{x^9-27}$ $\frac{x^6+3x^3+9 \leftarrow grouping\ (ac\ \&\ b)}{x^9-27 \leftarrow difference\ of\ cubes}$ $\frac{x^6+3x^3+9 \leftarrow not\ factorable}{(x^3-3)((x^3)^2+3x^3+(3)^2)}$ $\frac{x^6+3x^3+9}{(x^3-3)(x^6+3x^3+9)} \leftarrow cancel$ $\boxed{\frac{1}{x^3-3}}$	**EXAMPLE** Reduce the fraction as much as possible. $\frac{x^4-16}{5x^2+10}$ $\frac{x^4-16 \leftarrow difference\ of\ squares}{5x+10 \leftarrow both\ have\ 5s}$ $\frac{(x^2+4)(x^2-4) \leftarrow diff.squares}{5(x+2)}$ $\frac{(x^2+4)(x+2)(x-2)}{5(x+2)} \leftarrow cancel$ $\frac{(x^2+4)(x-2)}{5} \leftarrow simplify$ $\boxed{\frac{x^3-2x^2+4x-8}{5}}$
10. Reduce the fraction as much as possible. $\frac{x^{12}+8}{6x^4+12}$	11. Reduce the fraction as much as possible. $\frac{x^2-4}{3x^5+6x^4-4x-8}$	12. Reduce the fraction as much as possible. $\frac{x^9-343}{x^6-49}$
13. Reduce the fraction as much as possible. $\frac{5x^3+15x^2+2x+6}{x^2-9}$	14. Reduce the fraction as much as possible. $\frac{x^{27}+1}{x^{18}-1}$	15. Reduce the fraction as much as possible. $\frac{x^{10}-1}{3x^5+3}$