Name: _____ Per: ____

Rules of Logarithms

Logarithms are the **inverses** (opposites) of exponents, like multiplication is the inverse of division. Logarithms cancel out exponent bases, allowing you to solve for the exponent itself. Before we learn to solve logarithms, however, let's learn how to work with them. Logarithms follow a series of rules that work only for logarithms.

Rule #1: Logarithm bases cancel matching exponent bases, and exponent bases cancel matching logarithm bases.

EXAMPLE Write the exponent as a logarithm. $3^x = 7$ *The exponent base is 3, so $\log_3$ will cancel it. Remember to use it on both sides!* $\cancel{\log_3}\ \cancel{3}^x = \log_3 7$ $\boxed{x = \log_3 7}$	1. Write the exponent as a logarithm. $6^x = 5$	2. Write the exponent as a logarithm. $9^x = 4$
EXAMPLE Write the logarithm as an exponent. $\log_9 x = 2$ *The log base is 9, so an exponent base of 9 will cancel it. Remember to use it on both sides!* $\cancel{9}^{\cancel{\log_9} x} = 9^2$ $\boxed{x = 9^2}$	3. Write the logarithm as an exponent. $\log_2 x = 4$	4. Write the logarithm as an exponent. $\log_5 x = 3$

Rule #2: An **exponent inside** a logarithm gets **moved to the front**, where attaches **with multiplication**.

EXAMPLE Write the logarithm without an exponent. $\log_2(x^3)$ *Take the inside exponent off and move it to the front, where it multiplies.* $\log_2(x^3) = \boxed{3\log_2 x}$	5. Write the logarithm without an exponent. $\log_7(x^8)$	6. Write the logarithm without an exponent. $\log_3(x^5)$
EXAMPLE Write the logarithm without an exponent. $y = \log_9(x^5)$ *Take the inside exponent off and move it to the front, where it multiplies.* $y = \log_9(x^5)$ $y = \boxed{5\log_9 x}$	7. Write the logarithm without an exponent. $y = \log_6(x^2)$	8. Write the logarithm without an exponent. $y = \log_{11}(x^4)$

Rule #3: **Multiplication inside one** logarithm can be re-written as **addition between two** logarithms **of the same base**, and vice versa.

EXAMPLE Factor the inside and write as two logarithms (do not use 1 as a factor). $\log_5 6$ *Two numbers that **multiply** to 6 are: 2 & 3, so…* $\log_5 6 = \log_5(2 \cdot 3)$ *Now, split it into two **added** logs.* $\log_5(2 \cdot 3) = \boxed{\log_5 2 + \log_5 3}$	9. Factor the inside and write as two logarithms (do not use 1 as a factor). $\log_7 15$	10. Factor the inside and write as two logarithms (do not use 1 as a factor). $\log_2 21$

Rule #3 (cont'd):

EXAMPLE Write as one logarithm. $\log_5 2 + \log_5 6$ *They're adding and have the same base, so write it as one by multiplying the inside parts…* $\log_5 2 + \log_5 6 = \log_5(2 \cdot 6)$ $= \boxed{\log_5 12}$	11. Write as one logarithm. $\log_3 4 + \log_3 5$	12. Write as one logarithm. $\log_7 8 + \log_7 3$

Rule #4: **Division inside one** logarithm can be re-written as **subtraction between two** logarithms of the same base, and vice versa.

EXAMPLE Write as two logarithms. $\log_5\left(\frac{3}{2}\right)$ *Split into two (top first) and subtract them…*$\log_5\left(\frac{3}{2}\right) = \boxed{\log_5 3 - \log_5 2}$	13. Write as two logarithms. $\log_9\left(\frac{4}{7}\right)$	14. Write as two logarithms. $\log_3\left(\frac{2}{5}\right)$
EXAMPLE Write as one logarithm. $\log_3 6 - \log_3 12$ *They're dividing and have the same base, so write it as one by dividing the first inside by the second…* $\log_3 6 - \log_3 12 = \log_3\left(\frac{6}{12}\right)$ $= \boxed{\log_3 2}$	15. Write as one logarithm. $\log_8 10 - \log_8 15$	16. Write as one logarithm. $\log_5 18 - \log_5 6$

Rule #5: To use a calculator, you must change the base to 10 using the **change of base formula**.

EXAMPLE Change the base to 10. $\log_7 5$ *Create a fraction with $\log_{10}$ on the top and the bottom.* $\log_7 5 = \frac{\log_{10} \_\_}{\log_{10} \_\_}$ *The original log base (7) goes inside the bottom log (base on bottom); the original inside goes inside the top log:* $\log_7 5 = \boxed{\frac{\log_{10} 5}{\log_{10} 7}}$	17. Change the base to 10. $\log_3 2$	18. Change the base to 10. $\log_4 11$
EXAMPLE Change the base to 10. $\log_{12} 4$ *Create a fraction using $\log_{10}$: base on bottom, inside on top.* $\log_{12} 4 = \boxed{\frac{\log_{10} 4}{\log_{10} 12}}$	19. Change the base to 10. $\log_8 9$	20. Change the base to 10. $\log_6 3$