

## Evaluating Logarithms

Since logarithms are basically exponents written a new way, evaluating or solving them means that you have to answer the question: This base to **what power** (or exponent) will equal the inside?

For example: Evaluate  $\log_3 9$ .

The base of 3 to **what power** will equal the 9 inside? The answer is  $\boxed{2}$ , because  $3^{\boxed{2}} = 9$

1. Evaluate $\log_2 8$ .	2. Evaluate $\log_4 16$ .	3. Evaluate $\log_3 81$ .
4. Evaluate $\log_5 125$ .	5. Evaluate $\log_2 32$ .	6. Evaluate $\log_7 49$ .
7. Evaluate $\log_{12} 144$ .	8. Evaluate $\log_7 343$ .	9. Evaluate $\log_{10} 10,000$ .

The rules of logarithms are there to make logarithms easier to simplify and to evaluate.

<p><b>EXAMPLE</b></p> <p>Evaluate <math>\log_7 \left(\frac{1}{49}\right)</math>.</p> <p><i>A fraction is division, so I can split the log with division inside into two logs using subtraction (rule 4).</i></p> $\log_7 \left(\frac{1}{49}\right) = \log_7 1 - \log_7 49$ <p>Base 7 to <b>what power</b> will equal 1? <math>7^{\boxed{0}} = 1</math>.</p> <p>Base 7 to <b>what power</b> will equal 49? <math>7^{\boxed{2}} = 49</math>. So...</p> $\log_7 \left(\frac{1}{49}\right) = \log_7 1 - \log_7 49$ $= 0 - 2 = \boxed{-2}$	<p>10. Evaluate <math>\log_2 \left(\frac{1}{16}\right)</math>.</p>	<p>11. Evaluate <math>\log_5 \left(\frac{1}{625}\right)</math>.</p>
<p><b>EXAMPLE</b></p> <p>Evaluate <math>\log_3(243)</math>.</p> <p><i>When a number is too big, make it smaller. The base is 3, so use that...</i></p> $243 \div 3 = 81, \text{ so } (3)(81) = 243$ $\log_3(243) = \log_3((3)(81))$ <p><i>I can split 1 log with multiplication inside into 2 logs by addition (rule 3).</i></p> $\log_3(243) = \log_3((3)(81))$ $= \log_3(3) + \log_3(81)$ <p>Base 3 to <b>what power</b> will equal 3? <math>3^{\boxed{1}} = 3</math>.</p> <p>Base 3 to <b>what power</b> will equal 81? <math>3^{\boxed{4}} = 81</math>. So...</p> $= \log_3(3) + \log_3(81)$ $= 1 + 4 = \boxed{5}$	<p>12. Evaluate <math>\log_8 512</math>.</p>	<p>13. Evaluate <math>\log_5 3125</math>.</p>

<p><b>EXAMPLE</b> Evaluate <math>\log_4(16^5)</math>. <i>An exponent in a logarithm moves to the front and multiplies.</i> <math>\log_4(16^5) = 5 \log_4 16 = 5(2) = \boxed{10}</math></p>	14. Evaluate $\log_3(27^7)$ .	15. Evaluate $\log_2(32^9)$ .
<p><b>EXAMPLE</b> Evaluate <math>\log_6(216^{11})</math>. <math>\log_6(216^{11}) = 11 \log_6 216</math> <math>= 11(3) = \boxed{33}</math></p>	16. Evaluate $\log_8(64^3)$ .	17. Evaluate $\log_3(81^6)$ .

To determine the value of an exponent, you can use logarithms. To determine the value of more than one exponent, then you need to find a **common base** (a log base that is a base for both of the exponent bases), and use that as the base of the logarithm.

<p><b>EXAMPLE</b> Evaluate <math>4^x = 8^{x+3}</math>. <i>Find the common base of 4 &amp; 8...</i> <i>4 is a base for 4 (<math>4^1 = 4</math>),</i> <i>but not 8 (<math>4^{NOPE} = 8</math>).</i> <i>2 is a base for 4 (<math>2^2 = 4</math>),</i> <i>and 8 (<math>2^3 = 8</math>). So, use 2...</i> <math>4^x = 8^{x+3}</math> <math>\log_2(4^x) = \log_2(8^{x+3})</math> <i>Move the exponents forward...</i> <math>x \log_2 4 = (x + 3) \log_2 8</math> <i>Simplify the logarithms...</i> <math>x(2) = (x + 3)(3)</math> <math>2x = 3x + 9</math> <math>-x = 9</math> <math>x = \boxed{-9}</math></p>	18. Evaluate $27^x = 9^{x-5}$ .	19. Evaluate $125^x = 625^{x-5}$ .
<p><b>EXAMPLE</b> Evaluate <math>81^{x+7} = 27^{x-4}</math>. <i>Find the common base of 81 &amp; 27...</i> <i>9 is a base for 81 (<math>9^2 = 81</math>),</i> <i>but not 27 (<math>9^{NOPE} = 27</math>).</i> <i>3 is a base for 81 (<math>3^4 = 81</math>),</i> <i>and 27 (<math>3^3 = 27</math>). Use 3...</i> <math>81^{x+7} = 27^{x-4}</math> <math>\log_3(81^{x+7}) = \log_3(27^{x-4})</math> <i>Move the exponents forward...</i> <math>(x + 7) \log_3 81 = (x - 4) \log_3 27</math> <i>Simplify the logarithms...</i> <math>(x + 7)(4) = (x - 4)(3)</math> <math>4x + 28 = 3x - 12</math> <math>x + 28 = -12</math> <math>x = \boxed{-40}</math></p>	20. Evaluate $32^{x-2} = 4^{x-2}$ .	21. Evaluate $16^{x+8} = 64^{x-1}$ .