Name: ______________________________ Per: _______

Exponential Growth and Decay

Exponential growth is when a function is multiplied by a **constant ratio** greater than one.
For example: $f(x) = 7(1.5)^x$ *The constant ratio is 1.5.*
Exponential decay is when a function is multiplied by a **constant ratio** between zero and one.
For example: $f(x) = 2(0.3)^x$ *The constant ratio is 0.3.*

Does the expression show **growth** or **decay**?

1. $3(4.2)^x$	2. $400(0.12)^x$	3. $5(0.4)^x$
4. $7\left(\frac{4}{7}\right)^x$	5. $11\left(\frac{6}{5}\right)^x$	6. $210\left(\frac{2}{9}\right)^x$

Exponential growth and decay uses the formula:

$\boldsymbol{StartingAmount(1 \pm RateofChange)^{time}}$ which, when written technically, looks like this: $\boldsymbol{P(1 \pm r)^t}$

Rate of change is how much the amount **grew (increased means $1 + r$)** or **decayed (decreased means $1 - r$)**, and is usually written as a percent. To change the percent to a decimal, simply move the decimal point left two places.

EX Write as a decimal. 2.3% 02.3 → $\boxed{0.023}$	7. Write as a decimal. 3.6%	8. Write as a decimal. 15.6%	9. Write as a decimal. 7.4%

To write this expression from a word problem, there are a few things you need to identify: 1. The starting amount (P), 2. if it is using growth $(1 + r)$ or decay $(1 - r)$, 3. the percent change (as a decimal), and 4. the time that *is passing*.

EXAMPLE In 1994, the number of students in Holden High School was 760. If the annual rate of increase is about 1.2%, write an expression that represents the school population 9 years later. *Starting amount: 760 students* $\boxed{Growth(+)}$ *or decay? (says increase)* *Percent rate of change: 1.2%* *As a decimal: 0.012* *Amount of time passing: 9 years* $P(1 \pm r)^t = 760(1 + 0.012)^9$ $= \boxed{760(1.012)^9}$	10. The number of people who attended an annual conference in 2001 was 394. If the annual rate of decrease is about 11.3%, write an expression that represents the number of attendees 2 years later.	11. The average number of sales per month was 512 in 2007. If the annual rate of increase is about 3%, write an expression that represents the average number of sales 4 years later.
EXAMPLE In 1998, the principal balance was 394. If the annual rate of decrease is 14%, write an expression that represents the balance 6 years later. *Starting amount: 394* *Growth or* $\boxed{decay(-)}$*? (decrease)* *Percent rate of change: 14%* *As a decimal: 0.14* *Amount of time passing: 6years* $P(1 \pm r)^t = 394(1 - 0.14)^6$ $= \boxed{394(0.86)^6}$	12. In 2010, the population of a small town was 950. If the annual rate of increase is about 0.6%, write an expression to represent the population 3 years later.	13. In 1991, the average number of scrunchies a girl owned was 15. If the annual rate of decrease was 39%, write an equation to represent the average number of scrunchies 10 years later.

Now that you know how to create an exponential function from a word problem, the next step is to be able to recognize when a set of data is exponential and, if it is, to recognize the constant ratio.

If a set of data points is exponential, then you will be able to find a number that multiplies by each $f(x)$ coordinate to equal the next one. To determine if the set is exponential, first, determine the ratio between each of the f(x) terms and the term next to it (basically, divide them). It's easiest to start with the biggest and work your way to the smallest. Once you have each ratio, compare them. Are all of them the same? If yes, then it's exponential, and you've found the constant ratio. If no, then it's not exponential, so you're done.

For example: Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

x	-1	0	1	2
$f(x)$	3	6	12	24

Notice that the x is going up by 1 each time (there are no jumps)

Biggest f(x) is 24, followed by 12: $24 \div 12 = 2$ *ratio is 2.*
Next: 12 & 6: $12 \div 6 = 2$ *ratio is 2 again*
Next: $6 \div 3 = 2$ *ratio is 2!!!*
The ratios are all the same, so it is exponential, with a constant ratio of 2.

EXAMPLE
Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

x	-1	0	1	2
$f(x)$	0.4	1.2	2.4	4.8

Start with the biggest and divide: $4.8 \div 2.4 = 2$,
next: $2.4 \div 1.2 = 2$ *(so far, so good)*
next: $1.2 \div 0.4 = 3$ *THE RATIO CHANGED!!!*
It doesn't have a constant rate of change,
so it's not exponential.

EXAMPLE
Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

x	-1	0	1	2
$f(x)$	43.75	8.75	1.75	0.35

Start with the biggest and divide: $43.75 \div 8.75 = 5$,
next: $8.75 \div 1.75 = 5$ *(it's the same ratio)*
next: $1.75 \div 0.35 = 5$ *THE RATIO IS CONSTANT!!!*
It is exponential *with a* constant ratio of 5.

14. Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

x	-1	0	1	2
$f(x)$	67.2	22.4	5.6	1.4

15. Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

x	-1	0	1	2
$f(x)$	2.1	4.2	16.8	134.4

16. Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

x	-1	0	1	2
$f(x)$	8.4	4.2	0	-4.2

17 Determine whether f is an exponential function of x of the form $f(x) = ab^x$. If so, find the constant ratio.

x	-1	0	1	2
$f(x)$	1.7	10.2	61.2	367.2