

Variables in the Denominator

In this unit, we'll be working with **Rational Expressions**. Rational expressions are basically fractions that involve a variable. This means that we have to remember the rules of fractions and apply them to **rational expressions**.

Rule 1: To **add or subtract** fractions, they must have a **common denominator**. Then, **add or subtract the tops**.

To create a common denominator,

1) Factor the bottom of each fraction to see if any denominator factors on the fractions match.

(optional) Cancel any *not-matching* factors from each individual fraction.

2) Multiply the top and bottom of each fraction with any missing denominator factors
(factors that *the other fraction has but this fraction is does not have*).

<p>EXAMPLE</p> $\frac{1}{12} + \frac{2}{9}$ <p>1) Factor the bottoms to find matches...</p> $\frac{1}{12} + \frac{2}{9} = \frac{1}{(3)(4)} + \frac{2}{(3)(3)}$ <p><i>I could have factored the 12 as 2 & 6, but neither of those factors is also in 9, so I chose the factor set with a match.</i></p> <p>2) Multiply with the missing factors...</p> $\frac{1}{(3)(4)} + \frac{2}{(3)(3)}$ <p>Fraction 1 has 3 & 4 Fraction 2 has 3 & 3 Fraction 1 is missing the other 3 Fraction 2 is missing the 4</p> $\frac{1}{(3)(4)} \cdot \frac{3}{3} + \frac{2}{(3)(3)} \cdot \frac{4}{4}$ $\frac{(1)(3)}{(3)(4)(3)} + \frac{(2)(4)}{(3)(3)(4)}$ $\frac{3}{36} + \frac{8}{36} = \frac{11}{36}$	<p>1. $\frac{4}{35} - \frac{3}{28}$</p>	<p>2. $\frac{7}{16} + \frac{10}{24}$</p>
<p>EXAMPLE</p> $\frac{3}{2x} - \frac{5}{3x}$ <p>1) Factor the bottoms to find matches...</p> $\frac{3}{2x} - \frac{5}{3x} = \frac{3}{(2)(x)} - \frac{5}{(3)(x)}$ <p>2) Multiply with the missing factors...</p> <p>Fraction 1 has 2 & x Fraction 2 has 3 & x Fraction 1 is missing the 3 Fraction 2 is missing the 2</p> $\frac{3}{(2)(x)} \cdot \frac{3}{3} - \frac{5}{(3)(x)} \cdot \frac{2}{2}$ $\frac{(3)(3)}{(2)(x)(3)} - \frac{(5)(2)}{(3)(x)(2)}$ $\frac{9}{6x} - \frac{10}{6x} = \frac{-1}{6x}$	<p>3. $\frac{9}{5x} + \frac{6}{7x}$</p>	<p>4. $\frac{8}{9x} - \frac{1}{4x}$</p>

<p>EXAMPLE</p> $\frac{4}{5x+10} + \frac{2}{7x+14}$ <p>1) Factor the bottoms to <i>find matches</i>...</p> $\frac{4}{5(x+2)} + \frac{2}{7(x+2)}$ <p>2) Multiply with the <i>missing factors</i>...</p> <p>Fraction 1 is missing the 7 Fraction 2 is missing the 5</p> $\frac{4}{5(x+2)} \cdot \frac{7}{7} + \frac{2}{7(x+2)} \cdot \frac{5}{5}$ $\frac{(4)(7)}{(5)(x+2)(7)} + \frac{(2)(5)}{(7)(x+2)(5)}$ $\frac{28}{35x+70} + \frac{10}{35x+70} = \frac{38}{35x+70}$	<p>5. $\frac{3}{2x-2} + \frac{4}{3x-3}$</p>	<p>6. $\frac{7}{9x+27} - \frac{5}{6x+18}$</p>
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Rule 2: To multiply fractions, just multiply across the top and across the bottom (no common denominator needed).

<p>EXAMPLE</p> $\frac{3}{8} \cdot \frac{2}{7} = \frac{(3)(2)}{(8)(7)} = \frac{6}{56}$ <p>If you can reduce, you must.</p> $\frac{6}{56} = \frac{(3)(2)}{(28)(2)} = \frac{3}{28}$	<p>7. $\frac{9}{11} \cdot \frac{5}{6}$</p>	<p>8. $\frac{4}{15} \cdot \frac{3}{10}$</p>
<p>EXAMPLE</p> $\frac{6}{x} \cdot \frac{5}{9} = \frac{(6)(5)}{(x)(9)} = \frac{30}{9x}$ <p>REDUCE!!</p> $\frac{30}{9x} = \frac{(10)(3)}{(3x)(3)} = \frac{10}{3x}$	<p>9. $\frac{7}{8} \cdot \frac{10}{x}$</p>	<p>10. $\frac{15}{x} \cdot \frac{2}{25}$</p>
<p>EXAMPLE</p> $\frac{5}{x+3} \cdot \frac{3}{10} = \frac{(5)(3)}{(x+3)(10)} = \frac{15}{10x+30}$ <p>REDUCE!!</p> $\frac{15}{10x+30} = \frac{(3)(5)}{(2x+6)(5)} = \frac{3}{2x+6}$	<p>11. $\frac{12}{x-2} \cdot \frac{7}{16}$</p>	<p>12. $\frac{13}{x+1} \cdot \frac{2}{13}$</p>

Rule 3: To divide fractions, flip the second fraction and multiply across.

<p>EXAMPLE</p> $\frac{3}{8} \div \frac{9}{5}$ <p><i>Flip the second fraction and multiply.</i></p> $\frac{3}{8} \div \frac{9}{5} = \frac{3}{8} \cdot \frac{5}{9} = \frac{(3)(5)}{(8)(9)} = \frac{15}{72}$ $\frac{15}{72} = \frac{(5)(3)}{(24)(3)} = \frac{5}{24}$	<p>13. $\frac{2}{7} \div \frac{11}{21}$</p>	<p>14. $\frac{5}{12} \div \frac{5}{7}$</p>
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<p>EXAMPLE</p> $\frac{5}{7} \div \frac{x}{14}$ <p><i>Flip the second fraction and multiply.</i></p> $\frac{5}{7} \div \frac{x}{14} = \frac{5}{7} \cdot \frac{14}{x} = \frac{(5)(14)}{(7)(x)} = \frac{70}{7x}$ $\frac{70}{7x} = \frac{(10)(7)}{(x)(7)} = \frac{10}{x}$	<p>15. $\frac{6}{x} \div \frac{9}{10}$</p>	<p>16. $\frac{3}{2} \div \frac{33}{x}$</p>
<p>EXAMPLE</p> $\frac{9}{8} \div \frac{x+9}{2}$ <p><i>Flip the second fraction and multiply.</i></p> $\frac{9}{8} \div \frac{x+9}{2} = \frac{9}{8} \cdot \frac{2}{x+9} = \frac{(9)(2)}{(8)(x+9)}$ $= \frac{18}{8x+72} = \frac{(9)(2)}{(4x+36)(2)}$ $= \frac{9}{4x+36}$ <p><i>Remember, you can only cancel factors, which means you cannot cancel the 9 and the 36. 36 is not a factor of $4x + 36$!</i></p>	<p>17. $\frac{3}{x-2} \div \frac{6}{7}$</p>	<p>18. $\frac{15}{4} \div \frac{x+1}{2}$</p>

Rule 4: A fraction in a fraction is a division problem. Divide the top fraction by the bottom one and follow rule 3.

<p>EXAMPLE</p> $\frac{\frac{3}{8}}{\frac{9}{5}}$ <p><i>Write the double fraction as division.</i></p> $\frac{\frac{3}{8}}{\frac{9}{5}} = \frac{3}{8} \div \frac{9}{5}$ <p><i>Now, just divide.</i></p> $\frac{3}{8} \div \frac{9}{5} = \frac{3}{8} \cdot \frac{5}{9} = \frac{(3)(5)}{(8)(9)} = \frac{15}{72}$ $\frac{15}{72} = \frac{(5)(3)}{(24)(3)} = \frac{5}{24}$	<p>19. $\frac{\frac{2}{7}}{\frac{11}{21}}$</p>	<p>20. $\frac{\frac{5}{12}}{\frac{5}{7}}$</p>
<p>EXAMPLE</p> $\frac{\frac{5}{7}}{\frac{x}{14}}$ <p><i>Write the double fraction as division.</i></p> $\frac{\frac{5}{7}}{\frac{x}{14}} = \frac{5}{7} \div \frac{x}{14}$ <p><i>Divide.</i></p> $\frac{5}{7} \div \frac{x}{14} = \frac{5}{7} \cdot \frac{14}{x} = \frac{(5)(14)}{(7)(x)} = \frac{70}{7x}$ $\frac{70}{7x} = \frac{(10)(7)}{(x)(7)} = \frac{10}{x}$	<p>21. $\frac{\frac{6}{x}}{\frac{9}{10}}$</p>	<p>22. $\frac{\frac{3}{2}}{\frac{33}{x}}$</p>

EXAMPLE

$$\frac{\frac{9}{8}}{\frac{x+9}{2}}$$

Write the double fraction as division.

$$\frac{\frac{9}{8}}{\frac{x+9}{2}} = \frac{9}{8} \div \frac{x+9}{2}$$

Divide.

$$\begin{aligned} \frac{9}{8} \div \frac{x+9}{2} &= \frac{9}{8} \cdot \frac{2}{x+9} = \frac{(9)(2)}{(8)(x+9)} \\ &= \frac{18}{8x+72} = \frac{(9)(2)}{(4x+36)(2)} \\ &= \frac{9}{4x+36} \end{aligned}$$

23. $\frac{\frac{3}{x-2}}{\frac{6}{7}}$

24. $\frac{\frac{15}{4}}{\frac{x+1}{2}}$