

## Complex Fractions Involving Expressions

Sometimes, instead of simply dividing a fraction by a fraction, you have to divide a fraction problem by a fraction problem. As long as the denominator is a single fraction, all you have to do is flip the second fraction and multiply, using distribution when necessary.

**Simplify.****EXAMPLE**

$$\left(\frac{3}{x+1} + \frac{x+2}{5}\right) \div \frac{x+2}{x+1}$$

Since you're dividing by a single fraction, just flip it and multiply.

$$= \left(\frac{3}{x+1} + \frac{x+2}{5}\right) \cdot \frac{x+1}{x+2}$$

Distribute to each fraction inside.

$$= \left(\frac{3}{x+1}\right)\left(\frac{x+1}{x+2}\right) + \left(\frac{x+2}{5}\right)\left(\frac{x+1}{x+2}\right)$$

Cancel & simplify each.

$$= \frac{(3)(\cancel{x+1})}{(\cancel{x+1})(x+2)} + \frac{(\cancel{x+2})(x+1)}{(5)(\cancel{x+2})}$$

$$= \frac{3}{x+2} + \frac{x+1}{5}$$

Get a common denominator.

$$= \frac{3}{x+2} \cdot \frac{5}{5} + \frac{(x+1)}{5} \cdot \frac{(x+2)}{(x+2)}$$

$$= \frac{15}{5(x+2)} + \frac{(x+1)(x+2)}{5(x+2)}$$

Simplify by multiplying the factors.

$$= \frac{15}{5x+10} + \frac{x^2+3x+2}{5x+10}$$

Add.

$$= \frac{x^2+3x+17}{5x+10}$$

$$1. \left(\frac{13}{x-9} + \frac{x+3}{2}\right) \div \frac{x+3}{x-9}$$

$$2. \left(\frac{9}{x-2} - \frac{x+6}{4}\right) \div \frac{x+6}{x-2}$$

**EXAMPLE**

$$\left(\frac{11}{x+4} + \frac{x+8}{3}\right) \div \frac{x+8}{x+4}$$

Flip the dividing fraction. Distribute.

$$= \left(\frac{11}{x+4} + \frac{x+8}{3}\right) \cdot \frac{x+4}{x+8}$$

$$= \left(\frac{11}{x+4}\right)\left(\frac{x+4}{x+8}\right) + \left(\frac{x+8}{3}\right)\left(\frac{x+4}{x+8}\right)$$

$$= \frac{(11)(\cancel{x+4})}{(\cancel{x+4})(x+8)} + \frac{(\cancel{x+8})(x+4)}{(3)(\cancel{x+8})}$$

$$= \frac{11}{x+8} + \frac{x+4}{3}$$

$$= \frac{11}{x+8} \cdot \frac{3}{3} + \frac{(x+4)}{3} \cdot \frac{(x+8)}{(x+8)}$$

$$= \frac{33}{3(x+8)} + \frac{(x+4)(x+8)}{3(x+8)}$$

$$= \frac{33}{3x+24} + \frac{x^2+12x+32}{3x+24}$$

$$= \frac{x^2+12x+65}{3x+24}$$

$$3. \left(\frac{x-10}{6} - \frac{7}{x-5}\right) \div \frac{x-10}{x-5}$$

$$4. \left(\frac{6}{x-4} + \frac{x+3}{8}\right) \div \frac{x+3}{x-4}$$

**EXAMPLE**

$$\frac{\frac{x+1}{2} + \frac{7}{x-6}}{\frac{x+1}{x-6}}$$

Write as a division problem, and solve.

$$\begin{aligned} &= \left( \frac{x+1}{2} + \frac{7}{x-6} \right) \div \frac{x+1}{x-6} \\ &= \left( \frac{x+1}{2} + \frac{7}{x-6} \right) \cdot \frac{x-6}{x+1} \\ &= \left( \frac{x+1}{2} \right) \left( \frac{x-6}{x+1} \right) + \left( \frac{7}{x-6} \right) \left( \frac{x-6}{x+1} \right) \\ &= \frac{(x+1)(x-6)}{2(x+1)} + \frac{7(x-6)}{(x-6)(x+1)} \\ &= \frac{x-6}{2} + \frac{7}{x+1} \\ &= \frac{(x-6)(x+1)}{2(x+1)} + \frac{7}{(x+1)} \cdot \frac{2}{2} \\ &= \frac{(x-6)(x+1)}{2(x+1)} + \frac{(7)(2)}{2(x+1)} \\ &= \frac{x^2 - 5x - 6}{2x + 2} + \frac{14}{2x + 2} \\ &= \boxed{\frac{x^2 - 5x + 8}{2x + 2}} \end{aligned}$$

$$5. \frac{\frac{3}{x-4} - \frac{x+3}{12}}{\frac{x+3}{x-4}}$$

$$6. \frac{\frac{x+5}{4} + \frac{14}{x+6}}{\frac{x+5}{x+6}}$$

**EXAMPLE**

$$\frac{\frac{2}{x+6} - \frac{x-5}{11}}{\frac{x-5}{x+6}}$$

Write as a division problem, and solve.

$$\begin{aligned} &= \left( \frac{2}{x+6} - \frac{x-5}{11} \right) \div \frac{x-5}{x+6} \\ &= \left( \frac{2}{x+6} - \frac{x-5}{11} \right) \cdot \frac{x+6}{x-5} \\ &= \left( \frac{2}{x+6} \right) \left( \frac{x+6}{x-5} \right) - \left( \frac{x-5}{11} \right) \left( \frac{x+6}{x-5} \right) \\ &= \frac{2(x+6)}{(x+6)(x-5)} - \frac{(x-5)(x+6)}{11(x-5)} \\ &= \frac{2}{x-5} - \frac{x+6}{11} \\ &= \frac{2}{(x-5)} \cdot \frac{11}{11} - \frac{(x+6)(x-5)}{11(x-5)} \\ &= \frac{(2)(11)}{11(x-5)} - \frac{(x+6)(x-5)}{11(x-5)} \\ &= \frac{22}{11x-55} - \frac{x^2+x-30}{11x-55} \\ &= \boxed{\frac{-x^2-x+52}{11x-55}} \end{aligned}$$

$$7. \frac{\frac{x-5}{3} - \frac{4}{x+7}}{\frac{x-5}{x+7}}$$

$$8. \frac{\frac{x-10}{8} + \frac{6}{x+9}}{\frac{x-10}{x+9}}$$