

## Simplifying Rationals

You've learned how to simplify rational expressions when they've been factored for you. Today, you will follow the same process with one extra step: before you start, you need to factor the expressions in the numerator and the denominator.

**Simplify.**

<p><b>EXAMPLE</b></p> $\frac{19x^2 - 3x + 5}{8x - 13} - \frac{-7x^2 - 12x}{8x - 13}$ <p><i>They already have a common denominator... subtract across the top.</i></p> $\frac{(19x^2 - 3x + 5) - (-7x^2 - 12x)}{8x - 13}$ $\frac{19x^2 - 3x + 5 + 7x^2 + 12x}{8x - 13}$ $\frac{26x^2 + 9x + 5}{8x - 13}$ <p><i>Check to see if there are factors that can be canceled...the bottom can't be factored and neither can the top. SO...</i></p> $\boxed{\frac{26x^2 + 9x + 5}{8x - 13}}$	<p>1.</p> $\frac{-17x^2 + 10x + 9}{13x + 16} - \frac{14x^2 + 20}{13x + 16}$	<p>2.</p> $\frac{8x^2 + 16x - 1}{3x - 5} - \frac{17x^2 - 14x - 12}{3x - 5}$
<p><b>EXAMPLE</b></p> $\frac{x - 1}{x + 10} + \frac{x - 5}{x^2 + 13x + 30}$ <p><i>They need a common denominator.</i></p> <p><i>Factor the bottom of the 2<sup>nd</sup> fraction.</i></p> $x^2 + 13x + 30 \dots AC = 30, B = 13$ $3 \cdot 10 = 30, 3 + 10 = 13$ $x^2 + 3x + 10x + 30$ $= x(x + 3) + 10(x + 3)$ $= (x + 3)(x + 10)$ <p><i>Write it using the factors.</i></p> $\frac{x - 1}{x + 10} + \frac{x - 5}{(x + 3)(x + 10)}$ <p><i>1<sup>st</sup> fraction is missing (x+3)-multiply!</i></p> $\frac{(x + 3) \cdot (x - 1)}{(x + 3)(x + 10)} + \frac{x - 5}{(x + 3)(x + 10)}$ $\frac{x^2 + 4x + 3}{x^2 + 13x + 30} + \frac{x - 5}{x^2 + 13x + 30}$ $\frac{x^2 + 5x - 2}{x^2 + 13x - 30}$ <p><i>The top can't be factored, so there are no matching factors to cancel.</i></p> $\boxed{\frac{x^2 + 5x - 2}{x^2 + 13x - 30}}$	<p>3.</p> $\frac{x + 2}{x - 4} + \frac{5x + 15}{x^2 - x - 12}$	<p>4.</p> $\frac{x + 4}{x - 7} + \frac{3x + 18}{x^2 - x - 42}$

**EXAMPLE**

$$\frac{x^2 + 8x + 15}{x^2 + 7x + 10} \cdot \frac{x^2 + 6x + 8}{x^2 - 4x - 21}$$

Factor each part.

$x^2 + 8x + 15$ $AC = 15, B = 8$ $3 \& 5!$ $x^2 + 3x + 5x + 15$ $x(x+3) + 5(x+3)$ $(x+3)(x+5)$	$x^2 + 6x + 8$ $AC = 8, B = 6$ $2 \& 4!$ $x^2 + 2x + 4x + 8$ $x(x+2) + 4(x+2)$ $(x+2)(x+4)$
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$x^2 + 7x + 10$ $AC = 10, B = 7$ $2 \& 5!$ $x^2 + 2x + 5x + 10$ $x(x+2) + 5(x+2)$ $(x+5)(x+2)$	$x^2 - 4x - 21$ $AC = -21, B = -4$ $-7 \& 3!$ $x^2 - 7x + 3x - 21$ $x(x+7) + 3(x-7)$ $(x-7)(x+3)$
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Plug in the factors.

$$\frac{(x+3)(x+5)}{(x+5)(x+2)} \cdot \frac{(x+2)(x+4)}{(x-7)(x+3)}$$

Cancel and multiply.

$$\frac{\cancel{(x+3)}\cancel{(x+5)}}{\cancel{(x+5)}(x+2)} \cdot \frac{(x+2)(x+4)}{(x-7)\cancel{(x+3)}}$$

$$\frac{x+4}{x-7}$$

6.

$$\frac{x^2 - 3x}{x^2 - 18x + 45} \cdot \frac{x^2 - 11x - 60}{x^2 + 12x + 32}$$

7.

$$\frac{x^2 - 13x + 12}{x^2 - 11x - 12} \cdot \frac{x^2 + 11x + 10}{x^2 - 16x + 15}$$

**EXAMPLE**

$$\frac{x^2 - x - 6}{x^2 + x - 12} \div \frac{x^2 + 8x + 12}{x^2 + 9x + 20}$$

Flip the 2<sup>nd</sup> fraction.

$$\frac{x^2 - x - 6}{x^2 + x - 12} \cdot \frac{x^2 + 9x + 20}{x^2 + 8x + 12}$$

Factor and cancel.

$x^2 - x - 6$ $AC = -6, B = -1$ $-3 \& 2!$ $x^2 - 3x + 2x - 6$ $x(x-3) + 2(x-3)$ $(x-3)(x+2)$	$x^2 + 9x + 20$ $AC = 20, B = 9$ $4 \& 5!$ $x^2 + 4x + 5x + 20$ $x(x+4) + 5(x+4)$ $(x+4)(x+5)$
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$x^2 + x - 12$ $AC = -12, B = 1$ $-3 \& 4!$ $x^2 - 3x + 4x - 12$ $x(x-3) + 4(x-3)$ $(x+4)(x-3)$	$x^2 + 8x + 12$ $AC = 12, B = 8$ $2 \& 6!$ $x^2 + 2x + 6x + 12$ $x(x+2) + 6(x+2)$ $(x+2)(x+6)$
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$$\frac{(x-3)(x+2)}{(x+4)(x-3)} \cdot \frac{(x+4)(x+5)}{(x+2)(x+6)}$$

$$\frac{\cancel{(x-3)}\cancel{(x+2)}}{\cancel{(x+4)}(x-3)} \cdot \frac{\cancel{(x+4)}(x+5)}{\cancel{(x+2)}(x+6)}$$

$$\frac{x+5}{x+6}$$

8.

$$\frac{x^2 + x - 20}{x^2 - 5x - 24} \div \frac{x^2 + 17x + 60}{x^2 + 4x - 96}$$

9.

$$\frac{x^2 - 13x + 12}{x^2 - 3x + 2} \div \frac{x^2 - 20x + 96}{x^2 - x - 2}$$