Name: ______________________________

Rational Exponents and Radicals

As we know, the purpose of an exponent is to represent the number of times base number is multiplied by itself. A radical (or root) works the other direction. To solve a radical, you have to find the number that is being multiplied by itself (however many times the radical desires). Now, we're going to work with exponents and radicals at the same time through **rational exponents** (exponents that are fractions).

The numerator (or top) of the rational exponent works the way any exponent does—it makes the base bigger by telling you to multiply it over and over again. The denominator, however, is not the exponent you're used to having. It is actually a root, which means it makes the base smaller.

To solve a rational exponent problem, start with the denominator. Find the root (make the number smaller) before you start expanding it with the numerator (multiplying it by itself).

Simplify.

EXAMPLE $625^{\frac{3}{4}}$ *First, we'll use the denominator…* *Determine the 4th root of 625.* $625^{\frac{3}{4}} = \left(\sqrt[4]{625}\right)^3$ $5 \cdot 5 \cdot 5 \cdot 5 = 625$, *so the 4th root is 5* $\left(\sqrt[4]{625}\right)^3 = (5)^3$ *Now, use the exponent.* $(5)^3 = 5 \cdot 5 \cdot 5 = \boxed{125}$	**EXAMPLE** $16^{\frac{5}{2}}$ *Start with the denominator.* *Determine the 2nd root of 16.* $16^{\frac{5}{2}} = \left(\sqrt[2]{16}\right)^5$ $4 \cdot 4 = 16$, *so the 2nd root is 4* $\left(\sqrt[2]{16}\right)^5 = (4)^5$ *Now, use the exponent.* $(4)^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = \boxed{1024}$	**EXAMPLE** $343^{\frac{2}{3}}$ $343^{\frac{2}{3}} = \left(\sqrt[3]{343}\right)^2 = (7)^2 = \boxed{49}$
1. $8^{\frac{5}{3}}$	2. $216^{\frac{2}{3}}$	3. $64^{\frac{3}{2}}$
4. $243^{\frac{4}{5}}$	5. $729^{\frac{5}{6}}$	6. $81^{\frac{3}{2}}$
7. $27^{\frac{5}{3}}$	8. $256^{\frac{3}{4}}$	9. $32^{\frac{2}{5}}$

Sometimes, the radical is not written as a fractional exponent. When it's structured as a radical, remember that a radical is the same thing as a fractional exponent, which means that you can divide it out of any exponents that are hiding under that radical.

EXAMPLE $\sqrt[6]{729m^{12}n^{24}}$ *First, separate the numbers from the variables. We want our numbers to be under a radical. The variables, however, have exponents. These would be easier to solve as fractions.* $\sqrt[6]{729m^{12}n^{24}} = (\sqrt[6]{729})m^{\frac{12}{6}}n^{\frac{24}{6}}$ $= \boxed{3m^2n^4}$	**EXAMPLE** $\sqrt[3]{343x^9y^{30}z^3}$ $\sqrt[3]{343x^9y^{30}z^3} = (\sqrt[3]{343})x^{\left(\frac{9}{3}\right)}y^{\frac{30}{3}}z^{\frac{3}{3}}$ $= 7x^3y^{10}z^1$ $= \boxed{7x^3y^{10}z}$	**EXAMPLE** $\sqrt[5]{32a^{10}b^{-15}}$ $\sqrt[5]{32a^{10}b^{-15}} = (\sqrt[5]{32})a^{\frac{10}{5}}b^{-\frac{15}{5}}$ $= 2a^2b^{-3}$ $= \boxed{\frac{2a^2}{b^3}}$
10. $\sqrt[5]{1024d^{45}f^{70}}$	11. $\sqrt[3]{216p^{-6}q^{21}}$	12. $\sqrt[4]{625k^8m^{44}n^{32}}$
13. $\sqrt[3]{64r^3s^3t^{-3}}$	14. $\sqrt[4]{16a^{84}b^{12}}$	15. $\sqrt[5]{243c^{25}d^{35}}$
16. $\sqrt[3]{27x^9y^{36}}$	17. $\sqrt[4]{81g^{60}h^{-24}}$	18. $\sqrt[4]{256a^{48}b^{-20}c^{16}}$