Name: ____________________

Determining the Foci on Ellipses and Hyperbolas

You have learned how to identify all the parts of both the circle and the parabola. A circle is created with two things: a center and a radius. A parabola has three things: a center (the vertex), a directrix (the line), and a focus.

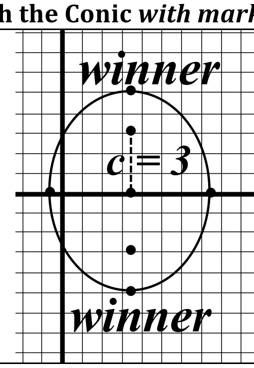
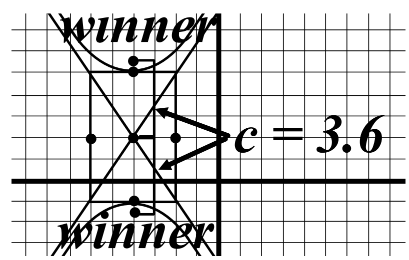
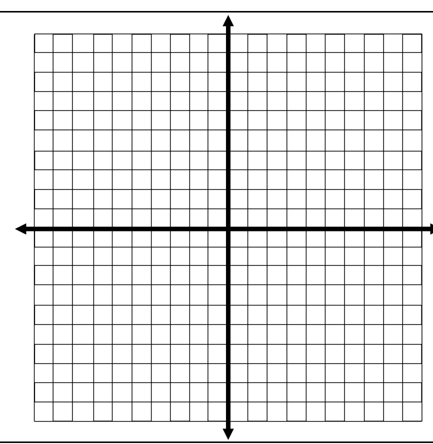
For ellipses and hyperbolas, you've learned how to determine the center, the major axis (the "winning" distance, a), which gives you the vertices, and the minor axis (the "losing" distance, b), which gives you the co-vertices. However, you have not yet learned how to determine the **foci** (more than one focus) **of ellipses and hyperbolas**. Today, you're going to learn how to do that from an equation, and then to graph each conic with its foci.

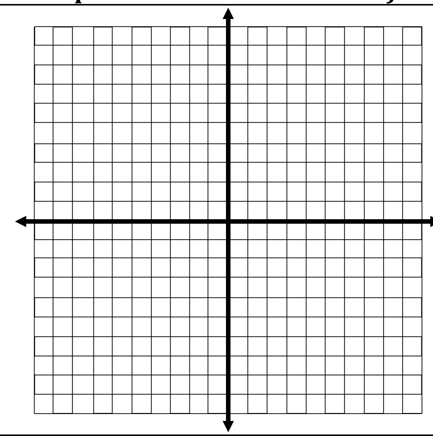
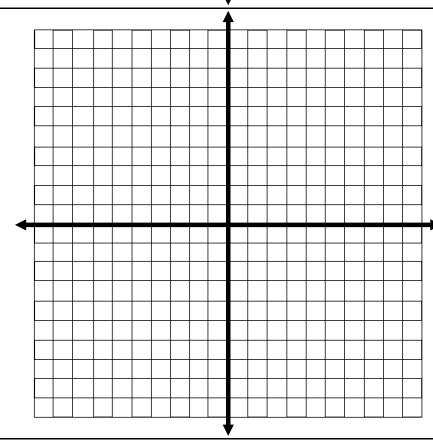
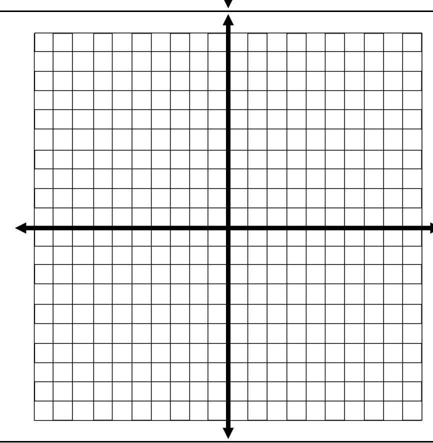
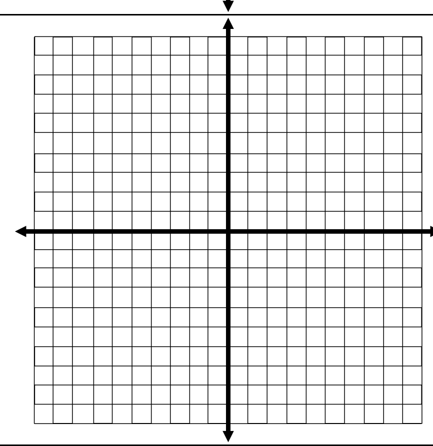
The distance from the center to the foci (which will always be on the major, or "winning", axis) is *c* on ellipses and hyperbolas. There is a simple formula to find it:

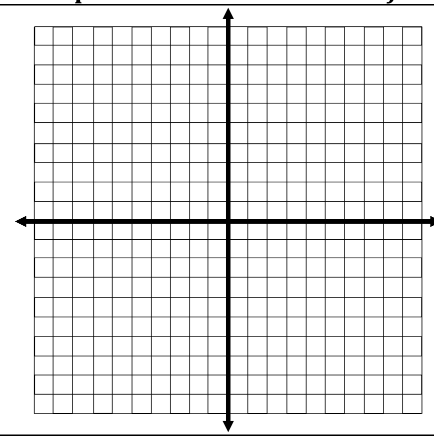
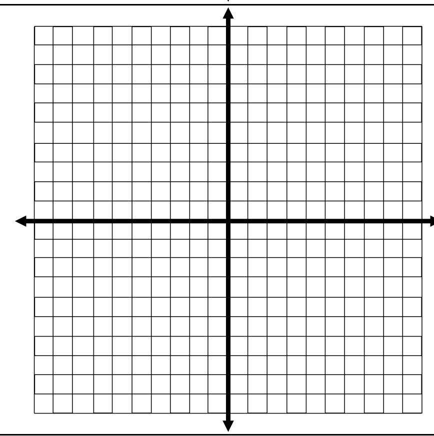
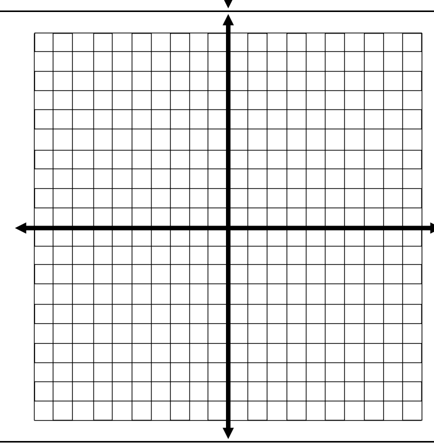
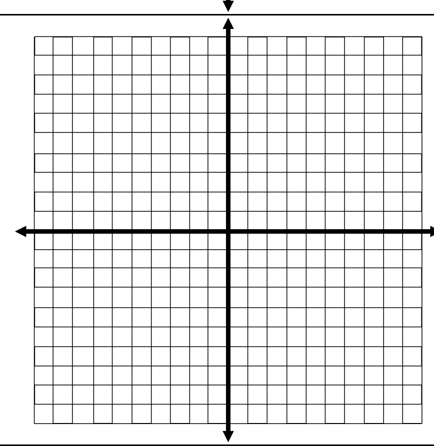
ELLIPSES $\rightarrow \boldsymbol{a^2 - b^2 = c^2}$

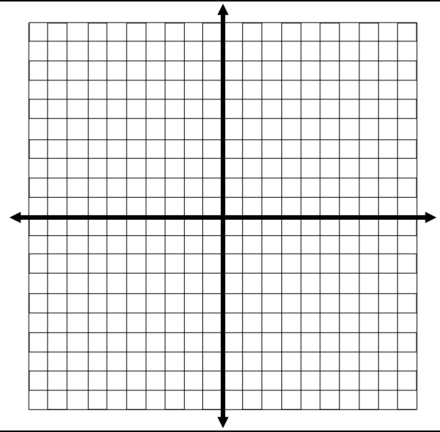
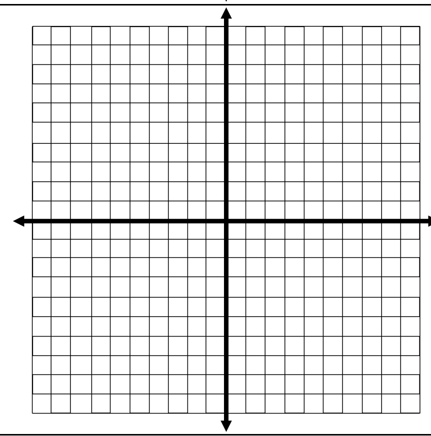
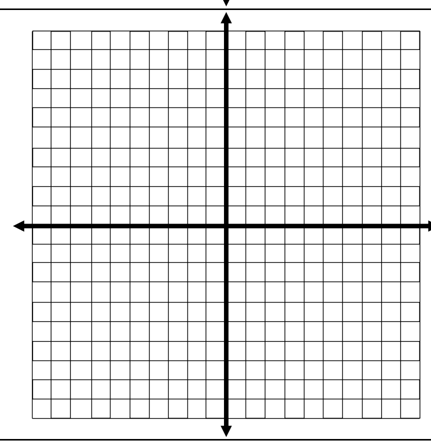
HYPERBOLAS $\rightarrow a^2 - (-b^2) = c^2$ or, simplified, $\rightarrow \boldsymbol{a^2 + b^2 = c^2}$

This formula comes from the Pythagorean theorem, which you learned in Geometry. Be careful, though. For conics, you don't **add** a^2 and b^2, you subtract them. It looks like adding for hyperbolas, but that is only because the "losing" denominator is negative.

Determine $(\boldsymbol{h}, \boldsymbol{k})$, $\boldsymbol{a}$ & $\boldsymbol{b}$	Determine $\boldsymbol{c}$	Graph the Conic *with marked foci*
EXAMPLE $\frac{(x-3)^2}{16} + \frac{y^2}{25} = 1$ $(h, k) = (3, 0)$ $a = \sqrt{25} = 5$ $b = \sqrt{16} = 4$	***Foci of an ELLIPSE:*** $a^2 - b^2 = c^2$ $a^2 = 25$ & $b^2 = 16$, so… $25 - 16 = c^2$ $9 = c^2$ $c = \sqrt{9} = \boxed{3}$	
EXAMPLE $\frac{(y-2)^2}{9} - \frac{(x+4)^2}{4} = 1$ $(h, k) = (-4, 2)$ $a = \sqrt{9} = 3$ $b = \sqrt{4} = 2$	***Foci of a HYPERBOLA:*** $a^2 - (-b^2) = c^2$ $a^2 + b^2 = c^2$ $a^2 = 9$ & $b^2 = 4$, so… $9 + 4 = c^2$ $13 = c^2$ $c = \sqrt{13}$ *13 is not perfect—there is no integer that multiplies by itself to get it. This means you must estimate. 13 is between 9 & 16.* $\sqrt{9} < \sqrt{13} < \sqrt{16}$, *so:* $3 < \sqrt{13} < 4$ *13 is just over halfway between 9 & 16, so it's about* $\boxed{3.6}$.	
1. $\frac{(x+1)^2}{36} + \frac{(y+1)^2}{64} = 1$		

Determine (h, k), a & b	Determine c	Graph the Conic *with marked foci*
2. $$\frac{(y+1)^2}{144} - \frac{(x-1)^2}{25} = 1$$		
3. $$\frac{(x+4)^2}{9} + \frac{(y-2)^2}{16} = 1$$		
4. $$\frac{(x+5)^2}{16} - \frac{(y-1)^2}{9} = 1$$		
5. $$\frac{(x+1)^2}{64} - \frac{(y-2)^2}{36} = 1$$		

Determine (h, k), a & b	Determine c	Graph the Conic *with marked foci*
6. $\frac{(x-2)^2}{100} + \frac{(y-2)^2}{36} = 1$		
7. $\frac{(x+2)^2}{25} + \frac{(y-1)^2}{1} = 1$		
8. $\frac{(y-5)^2}{16} - \frac{(x-3)^2}{9} = 1$		
9. $\frac{x^2}{144} + \frac{y^2}{169} = 1$		

Determine $(\boldsymbol{h}, \boldsymbol{k})$, $\boldsymbol{a}$ & $\boldsymbol{b}$	Determine $\boldsymbol{c}$	Graph the Conic ***with marked foci***
10. $\frac{(y+4)^2}{4} - \frac{(x+1)^2}{4} = 1$		
11. $\frac{(x+1)^2}{25} - \frac{y^2}{144} = 1$		
12. $\frac{x^2}{36} - \frac{(y-3)^2}{64} = 1$		
13. $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{36} = 1$