SEM 1: FINAL EXAM NOTES

QUADRATICS

I. Vertex

$+x^2$ minimum $-x^2$ maximum

A. From STANDARD form

$f(x) = ax^2 + bx + c$

Vertex: $x = \frac{-b}{2a}$

Vertex: y = plug that in!

$y = a\left(\frac{-b}{2a}\right)^2 + b\left(\frac{-b}{2a}\right) + c$

ex/ What is the absolute minimum of the function

Number only ↓

$f(x) = 1x^2 - 14x + 0$

$a = 1 \quad b = -14 \quad c = 0$

$x = \frac{-b}{2a} = \frac{-(-14)}{2(1)} = \frac{14}{2} = 7$

$y = 1(7)^2 - 14(7)$

$y = 49 - 98$

$y = -49$

vertex $(7, -49)$

Abs. min. is $(7, -49)$ $y = -49$

(for "Name the vertex" questions, do the same thing. Answer: (x, y))

B. Axis of Symmetry is the **X-VALUE** of the vertex

→ from 2 symmetrical points, find the MIDDLE of X

$\frac{x_1 + x_2}{2}$

ex/ Determine the axis of symm. if the x-int. are $(6, 0)$ & $(20, 0)$.

$\frac{x_1 + x_2}{2} = \frac{6 + 20}{2} = \frac{26}{2}$

$x = 13$

II. Direction of Parabola

$+x^2$ is UP $-x^2$ is DOWN

* ON THE TEST *

written: $-(-2x + 7)(5x - 4)$ ← multiplying

...MAKE it x^2

$-(-2x)(5x) = -(-10x^2)$

$= +10x^2$

$+x^2$ opens **UP**

III. Transformations in VERTEX form

$f(x) = a(x-h)^2 + k$

vertex: $(+h, +k)$ (h: opp, k: same)

1. $+a$ ~~NO Ref. over X~~
   $-a$ **Ref. over X**
2. a is Vertical Stretch (No sign in front)
3. Translations come from the vertex

ex/ How does this equation compare to the graph of $g(x) = x^2$?

$h(x) = -(x-4)^2 - 2$ (opp, same)

$-a$ means **Ref. over x**

CAN'T SEE a → NO VERT. STRETCH

VERTEX $(4, -2)$ — **Translation right 4 down 2**

ex/ $f(x) = 2(x-3)^2 + 5$

$+a$ NO Ref. over x

$a = 2$ — **Vertical Stretch by 2**

vertex $(+3, +5)$ — **Translates 3 right 5 up**

IV. Y-intercept

↳ in standard form

$f(x) = ax^2 + bx + c$

c is the y-int **(0, c)**

↳ in ANY form, plug in $x = 0$

ex/ Is $(0, 2)$ the y-int of the function $f(x) = 2x^2 - 4x - 1$? Why or Why NOT?

No, because c is -1.

or when I plug in 0,

$f(0) = 2(0)^2 - 4(0) - 1$ (y-int)

the y-int is $(0, -1)$.

V. Interval Solutions

	$\leq \geq$	$<>$
IN:	[zero, zero]	(zero, zero)
out:	$(-\infty, z] \cup [z, \infty)$	$(-\infty, z) \cup (z, \infty)$

*** ON Test ***

written: $ax^2 + bx + c \; \substack{\leq \\ \geq \\ < \\ >} \; 0$ — y

<0 shade above $\quad >0$ shade below

$\leq 0 \quad \geq 0$

TO TELL IF IT'S SHADED INSIDE OR OUTSIDE

ABOVE	BELOW	ABOVE	BELOW
UP	DOWN	DOWN	UP
≤ 0	≥ 0	≤ 0	≥ 0
above	below	ABOVE	BELOW
$+x^2$	$-x^2$	$-x^2$	$+x^2$
UP	DOWN	DOWN	UP

ex What are the interval solutions to $x^2 + 20 < 0$?

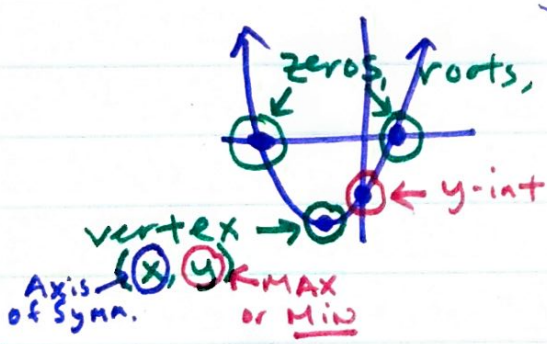
$x^2 + 9x$

[zero, zero] NO LINE UNDER $<$ $(-\infty, \text{zero}] \cup [\text{zero}, \infty)$

(zero, zero) <0 ABOVE $+x^2$ $(-\infty, z) \cup (z, \infty)$

≥ 0 BELOW $+x^2$

VI. Features on a graph



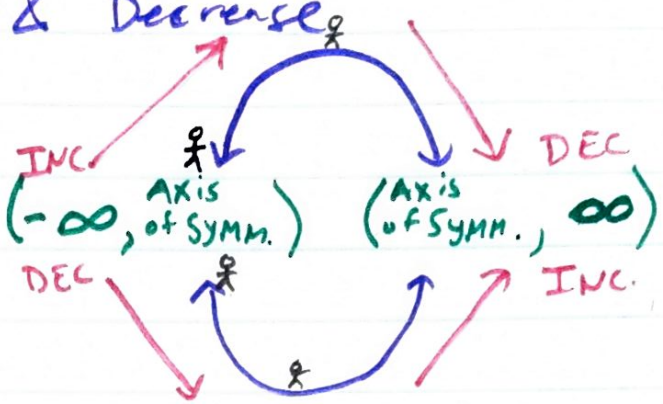
A. DOMAIN $(-\infty, \infty)$

B. Range

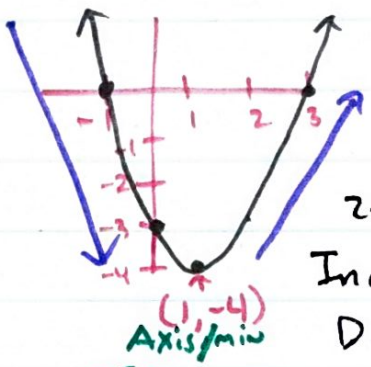
$[\text{min}, \infty)$ $y \geq \text{min}$

$(-\infty, \text{max}]$ $y \leq \text{max}$

C. Intervals of Increase & Decrease



ex



D: $(-\infty, \infty)$

R: $[-4, \infty)$

(-1, 0) (3, 0)

Zeros: $x = -1$ & $x = 3$

Inc: $(1, \infty)$

DEC: $(-\infty, 1)$

OF Symm

DEC $(-\infty, 1)$ INC. $(1, \infty)$

VII. Writing Zeros as factors

$f(x) = a(x \overset{\text{opp sign}}{z_1})(x \overset{\text{opp sign}}{z_2})$

ex What polynomial could have zeros at $(-5, 0)$ & $(2, 0)$?

$f(x) = \overset{?}{a}(x+5)(x-2)$

(A) $-3(x-2)(x+5)$

~~B $-5(x-5)(x+2)$~~

~~C $2(x-7)(x+5)$~~

(D) $(2x+10)(x-2)$

$2(x+5)(x-2)$

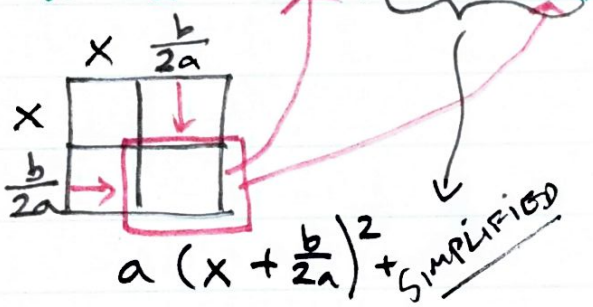
VIII. Finding zeros from equation

1. QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

2. Complete the Square

$a(x^2 + \frac{b}{a}x + \square) + c - a\square$



$a\left(x + \frac{b}{2a}\right)^2 +$ SIMPLIFIED

3. Factoring (by grouping)

* ON THE TEST *

4. Plug in the answers. Must = 0 to be correct.

ex/ What are the zeros of the quadratic function?

$f(x) = 3x^2 + 9x - 12$

Try $x=1$: $3(1)^2+9(1)-12 = 3+9-12 = 0$ ✓

A. $x=1$ or ~~$x=4$~~ Try $x=4$

B. ~~$x=-1$~~ or $x=4$ $3(4)^2+9(4)-12$

C. ~~$x=-1$~~ or $x=-4$ $3(16)+36-12$

D. $x=1$ or $x=-4$ $48+36-12=0$ ✓

ex/ Determine the x-int for $f(x) = 2x^2+x-28$

OPTION A: QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a=2 \quad b=1 \quad c=-28$

$$x = \frac{-(1) \pm \sqrt{(1)^2-4(2)(-28)}}{2(2)}$$

28 × 8 = 224

$$x = \frac{-1 \pm \sqrt{1-8(-28)}}{4} = \frac{-1 \pm \sqrt{1+224}}{4}$$

$$x = \frac{-1 \pm \sqrt{225}}{4} = \frac{-1 \pm 15}{4}$$

$\frac{-1+15}{4} = \frac{14}{4} = \frac{7}{2}$

$\frac{-1-15}{4} = \frac{-16}{4} = -4$

$(-4, 0)$ & $(\frac{7}{2}, 0)$

OPTION B: FACTORING

$f(x) = 2x^2+x-28$

$f(x) = 2x^2 - 7x + 8x - 28$

$A \cdot C = 2 \cdot -28 = -56 \quad B = 1$

mult. AC: −56; −7, +8; 1; b; add

$f(x) = 2x^2 - 7x + 8x - 28$

$x(2x-7) + 4(2x-7)$ — same

leftovers: $(x+4)(2x-7)$

$2(x+4)\left(x-\frac{7}{2}\right) = a(x\ )(x\ )$

$(-4, 0) \quad (\frac{7}{2}, 0)$

IX. Determining the number of REAL zeros

2 REAL zeros | 1 REAL zero | ∅ REAL zeros (2 imaginary)

A. Use PART of the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\pm\sqrt{+} = 2$ real $\pm\sqrt{-} = \varnothing$ real (2 imag)

$\pm\sqrt{0} = 1$ real

ex/ How many real solutions does the quadratic function $f(x) = x^2+4x+6$ have?

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(4) \pm \sqrt{(4)^2-4(1)(6)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16-24}}{2} = \frac{-4 \pm \sqrt{-8}}{2}$$ 2 imaginary answers

$\varnothing$ real

$$x = \frac{-4 \pm i\sqrt{8}}{2} = \frac{-4 \pm i\sqrt{4}\sqrt{2}}{2} = \frac{-4 \pm 2i\sqrt{2}}{2}$$

$x = -2 \pm i\sqrt{2}$ $(-2+i\sqrt{2}, 0)$

$(-2-i\sqrt{2}, 0)$

2 imag. No real

X. What completes the square? $+\boxed{?}$

A. Process:

$$ax^2+bx+c = 0$$

$$ax^2+bx = -c$$

$$a\left(x^2+\frac{b}{a}x+\square\right) = -c+a\square$$

$\frac{b}{a} \div 2$

$x \quad \frac{b}{2a}$

	x	$\frac{b}{2a}$
x	x^2	$\frac{b}{2a}x$
$\frac{b}{2a}$	$\frac{b}{2a}x$	$\left(\frac{b}{2a}\right)^2$

Shortcut: $+\left(\frac{b}{2a}\right)^2$

ex/ What correctly completes the square to solve the polynomial?

$$x^2+22x+\square = 32+\square$$

$$1\left(x^2+22x+\boxed{121}\right) = 32+\boxed{121}$$

$\div 2$

	x	$+11$
x	x^2	$11x$
$+11$	$11x$	121

shortcut: $\left(\frac{b}{2a}\right)^2 = \left(\frac{22}{2(1)}\right)^2 = (11)^2 = 121$

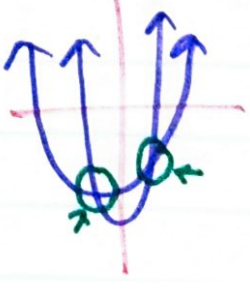
ex/ ... $x^2+6x+\boxed{9} = 19+\boxed{9}$

$$\left(\frac{b}{2a}\right)^2 = \left(\frac{6}{2(1)}\right)^2 = (3)^2 = 9$$

XI. Solve Systems

A. On a graph → where they cross

ex/ What are the solutions to the system of equations?



B. * ON THE TEST *

If you have equations & the answers... {

Plug in answers.

* Plugging in x MUST = y
* MUST work for ~~BOTH~~ equations

ex/ Which of the following are solutions to the system of equations? $\begin{cases} y = 2x - 2 \\ y = x^2 + 8x - 9 \end{cases}$

A. $(-7, -16)$ ~~$(3, 4)$~~ C. $(-7, -16)$ $(1, 0)$

B. $(7, 16)$ $(-1, 0)$ D. $(7, 16)$ $(-3, -4)$

Try $(-7, -16)$ $x = y$ ✓

$y = 2x - 2$
$y = 2(-7) - 2$
$y = -16$ ✓

$y = x^2 + 8x - 9$
$y = (-7)^2 + 8(-7) - 9$
$y = 49 - 56 - 9$
$y = -16$ ✓

Try $(3, 4)$ x y

$y = 2(3) - 2$
$y = 4$ ✓

$y = (3)^2 + 8(3) - 9$
$y = 9 + 24 - 9 = 24$

check $(1, 0)$

$y = 2(1) - 2$
$y = 0$ ✓

$y = (1)^2 + 8(1) - 9$
$y = 1 + 8 - 9$
$y = 0$ ✓

C. $(-7, -16)(1, 0)$

XII. Square Roots

A. Simplifying $\sqrt{\ }$

→ split $\sqrt{\ }$ into two $\sqrt{\ }$ factors
$\sqrt{\text{perfect}}\sqrt{\text{leftovers}}$

ex/ What is the equivalent to the radical expression?

$\sqrt{124}$

$\sqrt{4}\sqrt{31}$

$2\sqrt{31}$

124
~~1 124~~
~~2 62~~
~~4~~
4 · 31

B. Estimating $\sqrt{\ }$ s

→ look for two PERFECT SQUARES your number is between

ex/ Estimate $\sqrt{30}$

$\sqrt{25}$ $\sqrt{30}$ $\sqrt{36}$ $\sqrt{49}$

5 6 7

between 5 & 6

ex/ A ballroom is in the shape of square. The area is 131 square meters. The exact length of a side is between which two lengths?

$\sqrt{121}$ $\sqrt{131}$ $\sqrt{144}$

between 11 m & 12 m

x

$x^2 = 131\ m^2$

XIII. Inverses

A. Point

$(x, y) \rightarrow (y, x)$

ex/ ... inverse of ... $(-12, 4)$?

$\boxed{(4, -12)}$

B. Equation

$y = mx + b \rightarrow x = my + b$ (switch, then $-b$)

on test

$y = \frac{1}{m}x - \frac{b}{m}$

$f(x)$

$x - b = my$

$\frac{x-b}{m} = y$

ex/ what is the inverse of $f(x) = 9x + 17$?

$x = 9y + 17$

$x - 17 = 9y$

$\frac{x-17}{9} = y \rightarrow \boxed{f^{-1}(x) = \frac{1}{9}x - \frac{17}{9}}$

C. One-to-One

No x's repeat w/ diff y's

No y's repeat w/ diff x's

x	y
1	0
1	3
2	5

BAD

x	y
0	1
3	1
5	2

BAD

x	y
1	0
2	3
3	5

one-to-one goodness

ex/ $f(x) = 8x - 7$ one-to-one?

option 4:

x	y	
0	-7	$8(0) - 7$
1	1	$8(1) - 7$
2	9	$8(2) - 7$
3	17	$8(3) - 7$

$\boxed{\text{one-to-one}}$

ex/ ~~$f(x) = t$~~

Is $x = -11$ one-to-one?

x	y
-11	0
-11	1

$\boxed{\text{No}}$

XIV. Polynomials

A. Adding/Subtracting

→ Combine like terms

x^3 with x^3, x^2 with x^2, x with x & so on

* Don't forget to share the front sign *

ex/ Simplify

$+(x^5 - 4x) - (-2x^5 + 4x - 1)$

$+1x^5 - 4x + 2x^5 - 4x + 1$

$\boxed{3x^5 - 8x + 1}$

B. Product means MULTIPLICATION

C. "Show by drawing, an area model..." means MULTIPLY with

ex/ Show by drawing an **area model** or multiplication table how to find the **product** of $6x-3$ & $-4x+5$. Also, find the **product** of $(6x-3)$ & $-4x+5$.

	$(6x$	$-3)$
$(-4x$	$-24x^2$	$+12x$
$+5)$	$+30x$	-15

← 1 pt on final

rewr. then:

$-24x^2+12x+30x-15$

$\boxed{-24x^2+42x-15}$ ← 1pt on final

D. "Which correctly factors ___?"

option A: Factor it

option B: Multiply the answers to test them.