Name: _____ Per: ____

Determining the **Zeros** of a Quadratic
...aka the "roots," "solutions," or "x-intercepts"

The **zeros** *(roots/solutions/x-intercepts)* of a quadratic are simply the x-values when $y = 0$. This means that to find them, you have to plug in $y = 0$.

EXAMPLE: Determine the solutions of $f(x) = x^2 - 6x + 8$.

$$0 = x^2 - 6x + 8$$

Okay, we did that...now how do we solve for x? Well, there are three options:

FACTORING ***(Sometimes works)*** *What #'s multiply to c and add to b?*	COMPLETING THE SQUARE ***(Sometimes works easily)*** *Use the magic step*	QUADRATIC FORMULA ***(ALWAYS works)*** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Steps: $f(x) = ax^2 + bx + c$ 1. If a is not 1, <u>divide every term by a</u> ***(If not divisible, STOP!!! Use another easier way)*** 2. Puzzle out what 2 numbers multiply to c and add to b ***(If not possible, STOP!!! You <u>have to</u> use another way)*** 3. Write those numbers here→ $(x \_\_\_\_)(x\_\_\_\_)$ 4. Set it equal to zero 5. Set up two equations: $x \_\_ = 0$ *or* $x \_\_ = 0$ 6. Solve for your two x's	Steps: $f(x) = ax^2 + bx + c$ 1. Set the problem = 0 $ax^2 + bx + c = 0$ 2. If a is not 1, <u>divide every term by a</u> ***(If not divisible, STOP!!! Use another easier way)*** 3. Subtract the c term from both sides 4. Do the MAGIC STEP!!! ***(If not divisible by 2, STOP!!! Use another easier way)*** 5. Simplify the right side 6. Square root both sides—don't forget the ±! 7. Solve for x	Steps: $f(x) = ax^2 + bx + c$ 1. Determine a, b, & c. 2. Plug them into the equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 3. Simplify to determine x.
EXAMPLE: Determine the **solutions** of $f(x) = x^2 - 6x + 8$. $f(x) = x^2 - 6x + 8$ $(-2)(-4) = 12$ & $-2 + -4 = -7$ *Use -2 & -4* $f(x) = (x - 2)(x - 4)$ ←FACTORS $0 = (x - 2)(x - 4)$ $x - 2 = 0$ *or* $x - 4 = 0$ $x = 2$ *or* $x = 4$ $x = \boxed{\{2, 4\}}$ ←SOLUTIONS	**EXAMPLE:** Determine the **zeros** of $f(x) = x^2 - 6x + 8$. $x^2 - 6x + 8 = 0$ $x^2 - 6x = -8$ *MAGIC STEP!!!* $(x - 3)^2 = -8 + (-3)^2$ $(x - 3)^2 = -8 + 9$ $(x - 3)^2 = 1$ $\sqrt{(x - 3)^2} = \pm\sqrt{1}$ $x - 3 = \pm 1$ $x = 3 \pm 1$ $x = 3 + 1$ *or* $x = 3 - 1$ $x = 4$ *or* $x = 2$ $x = \boxed{\{2, 4\}}$ ←SOLUTIONS	**EXAMPLE:** Determine the **x-intercepts** of $f(x) = x^2 - 6x + 8$. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$ $x = \frac{6 \pm \sqrt{36 - 32}}{2} = \frac{6 \pm \sqrt{4}}{2} = \frac{6 \pm 2}{2}$ *SO...split it into two equations!* $x = \frac{6 - 2}{2} = \frac{4}{2} = 2$ *or* $x = \frac{6 + 2}{2} = \frac{8}{2} = 4$ $x = \boxed{\{2, 4\}}$ ←SOLUTIONS
EXAMPLE: Determine the **roots** of $f(x) = -2x^2 - 4x + 10$. $0 = -2x^2 - 4x + 10$ *÷ by* $a = -2$! $\frac{0}{-2} = \frac{-2x^2}{-2} - \frac{4x}{-2} + \frac{10}{-2}$ $0 = x^2 + 2x - 5$ $(1)(-5) = -5$, but $1 + -5 \neq 2$ $(-1)(5) = -5$, but $-1 + 5 \neq 2$ **NOT POSSIBLE—STOP!**	**EXAMPLE:** Determine the **solutions.** $f(x) = -2x^2 - 4x + 10$. $-2x^2 - 4x + 10 = 0$ *÷ by* $a = -2$! $\frac{-2x^2}{-2} - \frac{4x}{-2} + \frac{10}{-2} = \frac{0}{-2}$ $x^2 + 2x - 5 = 0$ *MAGIC STEP!!!* $x^2 + 2x = 5$ $(x + 1)^2 = 5 + (1)^2$ $(x + 1)^2 = 6$ $\sqrt{(x + 1)^2} = \pm\sqrt{6}$ $x + 1 = \pm\sqrt{6}$ $x = \boxed{\{-1 \pm \sqrt{6}\}}$	**EXAMPLE:** Determine the **zeros** of $f(x) = -2x^2 - 4x + 10$. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{4 \pm \sqrt{(-4)^2 - 4(-2)(10)}}{2(-2)}$ $x = \frac{4 \pm \sqrt{16 + 80}}{-4} = \frac{4 \pm \sqrt{96}}{-4}$ $x = \frac{4 \pm \sqrt{16}\sqrt{6}}{-4} = \frac{4 \pm 4\sqrt{6}}{-4}$ $x = \boxed{\{-1 \pm \sqrt{6}\}}$

1a. Determine the **roots** of $f(x) = x^2 - 8x + 16$	1b. Determine the **x-intercepts** of $f(x) = x^2 - 8x + 16$	1c. Determine the **solutions** of $f(x) = x^2 - 8x + 16$
2a. Determine the **zeros** of $f(x) = x^2 + 5x + 24$	2b. Determine the **roots** of $f(x) = x^2 + 5x + 24$	2c. Determine the **x-intercepts** of $f(x) = x^2 + 5x + 24$
3a. Determine the **solutions** of $f(x) = -3x^2 + 9x + 12$	3b. Determine the **zeros** of $f(x) = -3x^2 + 9x + 12$	3c. Determine the **roots** of $f(x) = -3x^2 + 9x + 12$
4a. Determine the **x-intercepts** of $f(x) = 2x^2 + x + 3$	4b. Determine the **solutions** of $f(x) = 2x^2 + x + 3$	4c. Determine the **zeros** of $f(x) = 2x^2 + x + 3$

5a. Determine the **roots** of $f(x) = 3x^2 - 2x + 6$	5b. Determine the **x-intercepts** of $f(x) = 3x^2 - 2x + 6$	5c. Determine the **solutions** of $f(x) = 3x^2 - 2x + 6$
6a. Determine the **zeros** of $f(x) = x^2 + 9x + 18$	6b. Determine the **roots** of $f(x) = x^2 + 9x + 18$	6c. Determine the **x-intercepts** of $f(x) = x^2 + 9x + 18$
7a. Determine the **solutions** of $f(x) = 5x^2 + 10x + 5$	7b. Determine the **zeros** of $f(x) = 5x^2 + 10x + 5$	7c. Determine the **roots** of $f(x) = 5x^2 + 10x + 5$
8a. Determine the **x-intercepts** of $f(x) = -x^2 - 4x + 3$	8b. Determine the **solutions** of $f(x) = -x^2 - 4x + 3$	8c. Determine the **zeros** of $f(x) = -x^2 - 4x + 3$

9a. Determine the **roots** of $f(x) = 4x^2 + 24x + 32$	9b. Determine the **x-intercepts** of $f(x) = 4x^2 + 24x + 32$	9c. Determine the **solutions** of $f(x) = 4x^2 + 24x + 32$
10a. Determine the **zeros** of $f(x) = -2x^2 + 12x - 10$	10b. Determine the **roots** of $f(x) = -2x^2 + 12x - 10$	10c. Determine the **x-intercepts** of $f(x) = -2x^2 + 12x - 10$

You can also determine solutions (zeros/roots/x-intercepts) in vertex form: $f(x) = a(x-h)^2 + k$. All you have to do is set the problem equal to 0 and solve for x.

EXAMPLE: Determine the **roots** of $f(x) = -3(x-2)^2 - 6$ $-3(x-2)^2 - 6 = 0$ $-3(x-2)^2 = +6$ $\frac{-3(x-2)^2}{-3} = \frac{6}{-3}$ $(x-2)^2 = -2$ $\sqrt{(x-2)^2} = \pm\sqrt{-2}$ $x - 2 = \pm i\sqrt{2}$ $x = \boxed{\{2 \pm i\sqrt{2}\}}$	11. Determine the **solutions** of $f(x) = 7(x+3)^2 - 21$	12. Determine the **zeros** of $f(x) = (x-9)^2 - 25$
EXAMPLE: Determine the **x-intercepts** of $f(x) = 4(x+6)^2 - 16$. $4(x+6)^2 - 16 = 0$ $4(x+6)^2 = 16$ $\frac{4(x+6)^2}{4} = \frac{16}{4}$ $(x+6)^2 = 4$ $\sqrt{(x+6)^2} = \pm\sqrt{4}$ $x - 2 = \pm 2$ $x = 2 \pm 2$ $x = 2 + 2 = 4$ *or* $x = 2 - 2 = 0$ $x = \boxed{\{0, 4\}}$	13. Determine the **zeros** of $f(x) = -5(x-2)^2 - 10$	14. Determine the **x-intercepts** of $f(x) = -(x+1)^2$