Name: ______________________________ Per: _____

Algebra 2: 1st Semester Benchmark Exam

Example Sheet 2

Study Guide Problem & Solution		New Example

Study Guide Problem & Solution

State whether the function has a maximum or minimum value and find it $f(x) = x^2 + 10x - 3$.

SINCE THE FIRST TERM IS POSITIVE (x^2), IT FACES UP. THAT MEANS IT HAS A MINIMUM. THE MIN IS THE Y-VALUE OF THE VERTEX.

$x = \frac{-b}{2a} = \frac{-10}{2(1)} = \frac{-10}{2} = -5$ PLUG IT IN TO FIND THE MIN @ Y!

$f(-5) = (-5)^2 + 10(-5) - 3 = 25 - 50 - 3 = -25 - 3 = -28$

THE MINIMUM IS AT Y = -28!

16 New Example: State whether the function has a maximum or minimum value and find it. $f(x) = -x^2 + 6x - 2$

Find the roots of the equation $14x - 60 = -2x^2$ by factoring.

ADD $-2x^2$ TO BOTH SIDES TO GET THE EQUATION IN STANDARD FORM.

$$2x^2 + 14x - 60 = 0$$

DIVIDE OUT THE TWO (BECAUSE x^2 SHOULD BE ALONE), THEN USE X-FACTOR!

$x^2 + 7x - 30 = 0$

$(x-3)(x+10) = 0$

$x - 3 = 0$ OR $x + 10 = 0$

$x = 3$ OR $x = -10$

$$\begin{array}{ccc} & -30 & \\ -3 & \times & 10 \\ & 7 & \end{array}$$

x = {-10, 3}

17 New Example: Find the roots of the equation $-15x + 90 = -5x^2$.

Write a quadratic function in standard form with zeros 3 and -2.

IF THE ZEROS ARE 3 & -2, THEN THAT MEANS THE FACTORS ARE (X-3) & (X – (-2)). SO, SET UP THE EQUATION AND MULTIPLY.

$f(x) = (x-3)(x+2)$

$f(x) = x^2 + 2x - 3x - 6$

$f(x) = x^2 - x - 6$

	x	+2
x	x^2	+2x
-3	-3x	-6

18 New Example: Write the quadratic function in standard form with zeros 5 and -7.

Given the equation $y = xn$ where $x > 1$ and $0 < n < 1$, which statement is valid for the real values of y?

A. $y < 0$ B. $y < x$ C. $y > x$ D. $y = 0$

PLUG IN EXAMPLES AND TRY TO ELIMINATE YOUR OPTIONS.

TRY $x = 2$ & $n = 0.5$.

$y = xn = 2(0.5)$

$y = 1$

SO THE ANSWER IS NOT A, SINCE Y > 0!

THE ANSWER IS NOT C, BECAUSE Y < X!

THE ANSWER IS NOT D, BECAUSE Y ≠ 0!

THE ANSWER MUST BE B. Y < X.

19 New Example: Given the equation $ax = by$ where $x > 0$, $a < -1$, $b > 1$, which statement is valid for the real values of y?

A. $y > 0$ B. $y < 0$ C. $y > x$ D. $y = 0$

Solve the equation $x^2 - 6x - 22 = 41$.

$x^2 - 6x - 22 = 41$ SUBTRACT 41 FROM BOTH SIDES

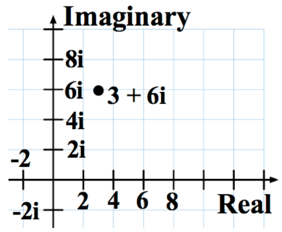
$x^2 - 6x - 63 = 0$ USE QUADRATIC FORMULA!!!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-63)}}{2(1)} = \frac{6 \pm \sqrt{36 + 252}}{2}$$

$$x = \frac{6 \pm \sqrt{288}}{2} = \frac{6 \pm \sqrt{144}\sqrt{2}}{2} = \frac{6 \pm 12\sqrt{2}}{2} = \frac{6}{2} \pm \frac{12\sqrt{2}}{2} = 3 \pm 6\sqrt{2}$$

THE SOLUTION IS $x = 3 \pm 6\sqrt{2}$!

20 New Example: Solve the equation $x^2 - 8x - 56 = 32$.

If x is a real number, which best describes the values of x for which the inequality $x^2 > 0$ is true? A. all $x < 0$ B. all $x \leq 0$ C. all values of x D. none **Since x^2 will ALWAYS equal a positive number, except when $x = 0$. The only way that $x^2 > 0$ is if $x \neq 0$. However, that is not one of our options, so I have to choose D. None.**	21	If x is a real number, which best describe the values of x for which the inequality $\frac{1}{x^2} > 0$ is true? A. $x < 0$ C. all values of x B. $x > 0$ D. all values of x, except when $x = 0$.
Express $5\sqrt{-117}$ in terms of i. **Test numbers to see if 117 has a perfect square factor.** **Since (9)(13)=117, I can simplify the radical:** $5\sqrt{-117} = 5\sqrt{-1}\sqrt{9}\sqrt{13} = (5)(3)i\sqrt{13} = \mathbf{15i\sqrt{13}}$	22	Express $2\sqrt{-176}$ in terms of i.
Find the complex conjugate of $7 - 2i$ **To create a complex conjugate, change the sign for the imaginary number.** <u>**7+2i**</u>	23	Find the complex conjugate of $3 + 4i$.
Graph the complex number $3 + 6i$.  	24	Graph the complex number $2 - 5i$.
Subtract. Write the result in the form $a + bi$. $(8 - 4i) - (2 + 3i)$ $8 - 4i - 2 - 3i =$ **$6 - 7i$**	25	Subtract. Write the result in the form $a + bi$. $(2 + 7i) - (3 - 6i)$
Multiply $4i(6 - 9i)$. Write the result in the form $a + bi$. **Distribute:** $24i - 36i^2 = 24i - 36(-1) = 24i + 36 =$ **$36 + 24i$**	26	Multiply $5i(9-3i)$. Write the result in the form $a + bi$.
Simplify $\frac{-5+9i}{3-3i}$ **Multiply the bottom's complex conjugate** $\left(\frac{-5+9i}{3-3i}\right)\left(\frac{3+3i}{3+3i}\right) = \frac{-15-15i+27i+27i^2}{9+9i-9i-9i^2} = \frac{-15+12i+27(-1)}{9-9(-1)}$ $= \frac{-15+12i-27}{9+9} = \frac{-42+12i}{18} = \frac{-42}{18} + \frac{12i}{18} = -\frac{7}{3} + \frac{2}{3}i$	27	Simplify $\frac{3-6i}{8+8i}$
A toy rocket is launched from the ground level with an initial vertical velocity 32 ft/s. The position of the rocket can be tracked using the following equation $f(t) = -16t^2 + 32t$, where t is the time in seconds. After how many seconds will the rocket hit the ground? **Find the zeros of the function:** $f(t) = -16t^2 + 32t$ $0 = -16t^2 + 32t$ **Divide by −16** $0 = t^2 - 2t$ **Factor out the** t $0 = t(t - 2)$ **Split up the equations & solve** $0 = t$ **or** $0 = t - 2$ $2 = t$ **The rocket will hit the ground at t = 2 seconds.**	28	A toy rocket is launched from the ground level with an initial vertical velocity of 48 ft/s. The position of the rocket can be tracked using the following equation $f(t) = -16t^2 + 48t$, where t is the time in seconds. After how many seconds will the rocket hit the ground?

Factor $x^3 + 3x^2 - 16x - 48$ completely.

A. $(x + 3)(x^2 + 16)$ C. $(x + 3)(x + 4)(x - 4)$

B. $(x - 3)(x^2 + 16)$ D. $(x - 3)(x + 4)(x - 4)$

THE EASIEST METHOD IS TO MULTIPLY EACH ANSWER CHOICE AND SEE WHICH ONE WORKS. REMEMBER, THOUGH—THE QUESTION ASKS FOR THE COMPLETE FACTORIZATION. MORE THAN ONE ANSWER COULD MULTIPLY TO MAKE $x^3 + 3x^2 - 16x - 48$. THE CORRECT ANSWER IS THE ONE THAT HAS THE MOST FACTORS.

(YOU COULD ALSO USE SYNTHETIC DIVISION TO NARROW YOUR CHOICES, IF YOU LIKE THAT METHOD BETTER)

I TESTED EACH ANSWER CHOICE. C IS THE CORRECT ANSWER.

C.

$(x+3)(x+4)(x-4)$

	x	+4
x	x^2	$\cancel{+4x}$
-4	$\cancel{-4x}$	-16

$(x+3)(x^2-16)$

	x	+3
x^2	x^3	$+3x^2$
-16	-16x	-48

$x^3+3x^2-16x-48$

THE FACTORS GIVEN FOR C MULTIPLY TO THE ORIGINAL EQUATION.

29 Factor $x^3 + 5x^2 - 49x - 245$ completely.

A. $(x + 5)(x^2 + 49)$ C. $(x + 5)(x + 7)(x - 7)$

B. $(x + 5)(x^2 - 49)$ D. $(x - 5)(x + 7)(x - 7)$

Divide.

$(x^2 - 4x + 7) \div (x + 3)$

USE EITHER SYNTHETIC OR LONG DIVISION. BOTH METHODS ARE SHOWN BELOW. USE WHICHEVER ONE YOU PREFER.

LONG DIVISION:

$x - 7 + \dfrac{28}{x+3}$

$x+3\,\overline{)\,x^2-4x+7}$ *$x(x)$ GIVES US x^2*

$-(x^2+3x)\ \downarrow$ *MULTIPLY $x(x+3)$, THEN SUBTRACT DOWN.*

$-7x+7$ *$(-7)(x)$ GIVES US $-7x$*

$-(-7x-21)$ *MULTIPLY $(-7)(x+3)$, THEN SUBTRACT.*

28 *THIS IS THE REMAINDER: PUT IT OVER $x+3$*

SOLUTION: $x - 7 + \dfrac{28}{x+3}$

SYNTHETIC DIVISION:

(x+3) IS A LIAR, SO WE PUT −3 IN THE BOX.

THE COEFFICIENTS ARE $1x^2 - 4x + 7$

$\boxed{-3}$	1	−4	7
	↓	−3	21
	1	−7	$\boxed{28}$
	x	#	R

SOLUTION: $x - 7 + \dfrac{28}{x+3}$

30 Divide.

$(x^2 + 2x - 6) \div (x - 4)$