Definition of Congruence $(≅)$

Today, we’re going to add the **Definition of Congruence** to our list of proof properties and theorems. But first, we need to understand the term “congruence.” The word “congruent” means that figures are the same shape and have the same size. Congruent and Equal are very similar. **Equal** is specifically that they have the **same size**, but **congruent** $(≅)$ is that they are the **same shape** and have the **same size**.

So, if two of the same shapes are congruent, then their sizes are equal. It also works the other way: if two of the same shapes have equal sizes, then they are congruent.

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| **Definition of Congruence**…means that two of the same shapes that are congruent will have equal sizes. | **Definition of Congruence** …means that two of the same shapes that have equal sizes are congruent. |
| $$Same shape≅Same size$$$$Same shape=Same size$$ | $$Same shape=Same size$$$$Same shape≅Same size$$ |
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| Example: |  |
| Given: $∠ABC≅∠DEF$ Prove: $m∠ABC=m∠DEF$ |
| $∠ABC≅∠DEF$  | Given |
| $$m∠ABC=m∠DEF$$ | **Definition of Congruence** |

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| Example: |  |
| Given: $m∠ABC=m∠DEF$Prove: $∠ABC≅∠DEF$ |
| $$m∠ABC=m∠DEF$$ | Given |
| $∠ABC≅∠DEF$  | **Definition of Congruence** |

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**Fill in the blanks in the proofs below.**

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| 1. |  |  | 2. |  |
| Given: $m∠1=m∠2$Prove: $∠1≅∠2$ |  | Given: $∠T≅∠V$Prove: $m∠T=m∠V$ |  |
| $$m∠1=m∠2$$ |  |  | $$∠T≅∠V$$ |  |
| $$∠1≅∠2$$ |  |  | $$m∠T=m∠V$$ |  |

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| 3. |  |  | 4. |  |
| Given: $\overbar{BC}≅\overbar{CD}$Prove: $BC=CD$ |  | Given: $NP=YZ$Prove: $\overbar{NP}≅\overbar{YZ}$ |  |
| $$\overbar{BC}≅\overbar{CD}$$ |  |  | $$NP=YZ$$ |  |
| $$BC=CD$$ |  |  | $$\overbar{NP}≅\overbar{YZ}$$ |  |

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| 5. |  |  | 6. |  |
| Given: $m∠LMN=17, m∠PQR=17$Prove: $∠LMN≅∠PQR$ |  | Given: $\overbar{AB}≅\overbar{GH}$, $AB=8$Prove: $GH=8$ |  |
| $$m∠LMN=17, $$$$m∠PQR=17$$ |  |  | $$\overbar{AB}≅\overbar{GH}$$ |  |
| $$17=17$$ |  |  | $$AB=GH$$ |  |
| $$m∠LMN= m∠PQR$$ |  |  | $$AB=8$$ |  |
| $$∠LMN≅∠PQR$$ |  |  | $$8=GH$$ |  |
|  |  |  | $$GH=8$$ |  |

Vertical Angles Theorem

Before we talk about the Vertical Angles Theorem, let’s take a look at some linear pair problems, involving two crossed lines.

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| 1. 1a. $∠CEB \& ∠BEA$ are a linear pair. $m∠CEB=83˚.$ $m∠BEA= ?$1b. You just figured out $m∠BEA$. $∠BEA and ∠AED$ are a linear pair. $m∠AED= ?$1c. You just figured out $m∠AED$. $∠AED and ∠DEC$ are a linear pair. $m∠DEC= ?$  | 2.2a. $∠FEJ \& ∠JEH$ are a linear pair. $m∠FEJ=111˚.$ $m∠JEH= ?$2b. You just figured out $m∠JEH$. $∠JEH and ∠HEG$ are a linear pair. $m∠HEG= ?$2c. You just figured out $m∠HEG$. $∠HEG and ∠GEF$ are a linear pair. $m∠GEF= ?$  | 3.3a. $∠KLM \& ∠MLN$ are a linear pair. $m∠KLM=74˚.$ $m∠MLN= ?$3b. You just figured out $m∠MLN$. $∠MLN and ∠NLP$ are a linear pair. $m∠NLP= ?$3c. You just figured out $m∠NLP$. $∠NLP and ∠PLK are a linear pair. $$m∠PLK= ?$  |

**What do you notice about the angle pairs that are located across the X from each other (not next to each other like the linear pairs)?**

The **Vertical Angles Theorem** tells us that, if two angles are vertical, then they are congruent to each other.

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| **Vertical Angles Theorem**…means that if two angles are vertical (across an X from each other), then the angles are congruent. |
| $$The angles are vertical$$$$∠Vertical Angles ≅∠Theorem$$ |
| Example:

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| Given: $∠A and ∠B are vertical angles$Prove: $∠A≅∠B$ |
| $$∠A and ∠B are vertical angles$$ | Given |
| $$∠A≅∠B$$ | **Vertical Angles Theorem** |

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Fill in the blanks in the proofs below.

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| 4. |  |  | 5. |  |
| Given: $∠2 \& ∠5 are vertical angles$Prove: $m∠2=m∠5$ |  | Given: Prove: $m∠ACB=m∠ECD$ |
| $$∠2 \& ∠5 are vertical angles$$ |  |  | $∠ACB \& ∠ECD$ are vertical angles |  |
| $$∠2≅∠5$$ |  |  | $$∠ACB≅∠ECD$$ |  |
| $$m∠2=m∠5$$ |  |  | $$m∠ACB=m∠ECD$$ |  |

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| 6. |  |  | 7. |  |
| Given: Prove: $m∠MLK=50˚$ |  |  Given: $∠DEF \& ∠RST $are vertical anglesProve: $m∠2=m∠5$ |  |
| $∠MLK and ∠PLN$ are vertical angles |  |  | $∠DEF \& ∠RST $are vertical angles |  |
| $$∠MLK≅∠PLN$$ |  |  | $$∠DEF≅∠RST$$ |  |
| $$m∠MLK=m∠PLN$$ |  |  | $$m∠DEF=m∠RST$$ |  |
| $$m∠PLN=50˚$$ |  |  |  |  |
| $$m∠MLK=50˚$$ |  |  |  |  |