Side-Angle-Side Triangle Congruence (SAS)

There are 5 properties that are used to prove triangles congruent: **SSS,** **SAS,** ASA, AAS, and HL. We have already talked about SSS. Today, we are going to discuss **SAS**. Side-Angle-Side means that if two sides of a triangle and the angle that sits between them are congruent to the two sides and the angle that sits between them on another triangle, then the triangles are congruent. So, if the side, then angle, then side are the same on the two triangles, then the triangles are the same.

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| **Side-Angle-Side (SAS) Triangle Congruence** | **Example proof using SAS** |
| **Setup steps:** Side1 on Triangle1 is congruent to Side1 on Triangle2 Side2 on Triangle1 is congruent to  Side2 on Triangle2 Angle between Sides1&2 on Triangle1 is congruent to  Angle between Sides1&2 on Triangle2**Resulting step:** The triangles are congruent by **SAS**. |

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| Given: $\overbar{AB}≅\overbar{DF}, ∠B≅∠F, \overbar{BC}≅\overbar{FE}$ Prove: $△ABC≅△DFE$ |
| Statements | Reasons |
| $$\overbar{AB}≅\overbar{DF}$$ | Given |
| $$∠B≅∠F$$ | Given |
| $$\overbar{BC}≅\overbar{FE}$$ | Given |
| $$△ABC≅△DFE$$ | Side-Angle-Side (SAS) |

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Fill in the reasons on the proofs below. Be careful, though, you now know 2 ways to prove that triangles are congruent.

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| 1. |  | 2. |
| Given:  Prove: $△MLK≅△QPN$ |  | Given: $\overbar{EC}≅\overbar{JH}, ∠C≅∠H, and \overbar{CA}≅\overbar{HF}$ Prove: $△AEC≅△FJH$ |
| Statements | Reasons |  | Statements | Reasons |
| $$\overbar{ML}≅\overbar{QP}, ∠L≅∠P, $$$$and \overbar{LK}≅\overbar{PN}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{EC}≅\overbar{JH}, ∠C≅∠H, $$$$and \overbar{CA}≅\overbar{HF}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△MLK≅△QPN$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△AEC≅△FJH$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| 3. |  | 4. |
| Given: $R is the midpoint of \overbar{MQ}$ Prove: $△MNR≅△QPR$ |  | Given:   Prove: $△TSV≅△VWT$ |
| Statements | Reasons |  | Statements | Reasons |
| $∠NRM and ∠PRQ$ are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$∠s$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{SV}≅\overbar{WT}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠NRM≅∠PRQ$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{SV}||\overbar{WT}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$R is the midpoint of \overbar{MQ}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $∠WTV$ and $∠SVT$ are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ $∠s$  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$\overbar{MR}≅\overbar{QR}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠WTV≅∠SVT$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$\overbar{NR}≅\overbar{PR}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{TV}≅\overbar{TV}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△MNR≅△QPR$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△TSV≅△VWT$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| 5. |  | 6. |
| Given:  Prove: $△BCD≅△GFE$ |  | Given: $H is the midpoint of \overbar{GK}$ Prove: $△GLH≅△KLH$ |
| Statements | Reasons |  | Statements | Reasons |
| $$BC=14, GF=14, $$$$m∠B=12˚, m∠G=12˚ $$$$DB=11, EG=11$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$H is the midpoint of \overbar{GK}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$14=14$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{GH}≅\overbar{KH}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$BC=GF$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{LH}≅\overbar{LH}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$\overbar{BC}≅\overbar{GF}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠LHK=90˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$12˚=12˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $∠LHK$ and $∠LHG$ are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ $∠s$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠B=m∠G$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠LHK+m∠LHG=180˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠B≅∠G$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$90˚+m∠LHG=180˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$11=11$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠LHG=90˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$DB=EG$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$90˚=90˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$\overbar{DB}≅\overbar{EG}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠LHK=m∠LHG$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△BCD≅△GFE$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△GLH≅△KLH$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| 7. |  | **PROOF 7 Continued** |
| Given: $x=4$  Prove: $△GHJ≅△LKM$ |  | $$LK=6\left(4\right)-4$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$LK=20$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$GH=LK$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$\overbar{GH}≅\overbar{LK}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Statements | Reasons |  | $$JG=5\left(4\right)-6$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠G=y˚, m∠L=y˚, $$$$GH=5x, $$$$JG=5x-6, $$$$LK=6x-4, $$$$ML=3x+2, \& x=4$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$JG=14$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$ML=3\left(4\right)+2$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$y˚=y˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$ML=14$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠G=m∠L$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$JG=ML$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠G≅∠L$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{JG}≅\overbar{ML}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$GH=5(4)$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△GHJ≅△LKM$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$GH=20$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  |  |  |
| 8. |  | 9. |
| Given: $\overbar{RP}≅\overbar{SP}, \overbar{PT}≅\overbar{PT}, and \overbar{TR}≅\overbar{TS}$ Prove: $△RPT≅△SPT$ |  | Given: $\overbar{MN}≅\overbar{PQ}, ∠N≅∠Q, and \overbar{NL}≅\overbar{QN}$ Prove: $△MLN≅△PNQ$ |
| Statements | Reasons |  | Statements | Reasons |
| $$\overbar{RP}≅\overbar{SP}, \overbar{PT}≅\overbar{PT}, $$$$and \overbar{TR}≅\overbar{TS}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{MN}≅\overbar{PQ}, ∠N≅∠Q, $$$$and \overbar{NL}≅\overbar{QN}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△RPT≅△SPT$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△MLN≅△PNQ$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| Given: $AB=4x+3, CB=7x-6, x=3, CD=y+5, AD=2y, y=5, m∠ABD=22˚, m∠CBD=22˚$ Prove: $△ABD≅△CBD$ |
| Statements | Reasons |
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