Angle-Side-Angle Triangle Congruence (ASA)

There are 5 properties that are used to prove triangles congruent: **SSS,** **SAS,** **ASA,** AAS, and HL. We have already talked about SSS & SAS. Today, we are going to discuss **ASA**. Angle-Side-Angle means that if two angles of a triangle and the side that sits between them are congruent to the two angles and the side that sits between them on another triangle, then the triangles are congruent. So, if the angle, then side, then angle are the same on the two triangles, then the triangles are the same.

|  |  |
| --- | --- |
| **Angle-Side-Angle (ASA) Triangle Congruence** | **Example proof using ASA** |
| **Setup steps:** Angle1 on Triangle1 is congruent to Angle1 on Triangle2 Angle2 on Triangle1 is congruent to  Angle2 on Triangle2 Side between Angles1&2 on Triangle1 is congruent to  Side between Angles1&2 on Triangle2**Resulting step:** The triangles are congruent by **ASA**. |

|  |
| --- |
| Given: $∠B≅∠F, \overbar{BC}≅\overbar{FE}, ∠C≅∠E$ Prove: $△ABC≅△DFE$ |
| Statements | Reasons |
| $$∠B≅∠F$$ | Given |
| $$\overbar{BC}≅\overbar{FE}$$ | Given |
| $$∠C≅∠E$$ | Given |
| $$△ABC≅△DFE$$ | Angle-Side-Angle (ASA) |

 |

Fill in the reasons on the proofs below. Be careful, though, you now know 2 ways to prove that triangles are congruent.

|  |  |  |
| --- | --- | --- |
| 1. |  | 2. |
| Given:  Prove: $△MLK≅△QPN$ |  | Given: $∠E≅∠J, \overbar{EC}≅\overbar{JH}, and ∠C≅∠H$ Prove: $△AEC≅△FJH$ |
| Statements | Reasons |  | Statements | Reasons |
| $$∠M≅∠Q, \overbar{ML}≅\overbar{QP}, $$$$∠L≅∠P$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠E≅∠J, \overbar{EC}≅\overbar{JH}, $$$$and ∠C≅∠H$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△MLK≅△QPN$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△AEC≅△FJH$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

|  |  |  |
| --- | --- | --- |
| 3. |  | 4. |
| Given: $R is the midpoint of \overbar{MQ}$ Prove: $△MNR≅△QPR$ |  | Given:  Prove: $△TSV≅△VWT$ |
| Statements | Reasons |  | Statements | Reasons |
| $∠NRM and ∠PRQ$ are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$∠s$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $∠SVT$ and $∠WTV$ are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ $∠s$  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠NRM≅∠PRQ$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠SVT≅∠WTV$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$R is the midpoint of \overbar{MQ}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{SV}≅\overbar{WT}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$\overbar{MR}≅\overbar{QR}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{SV}||\overbar{WT}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $∠MNR and ∠QPR$ are \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$∠s$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{TV}≅\overbar{TV}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠MNR≅∠QPR$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠TSV≅∠VWT$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△MNR≅△QPR$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△TSV≅△VWT$$ |  |

|  |  |  |
| --- | --- | --- |
| 5. |  | 6. |
| Given:  Prove: $△BCD≅△GFE$ |  | Given: $H is the midpoint of \overbar{GK}, x=7$ Prove: $△GLH≅△KLH$ |
| Statements | Reasons |  | Statements | Reasons |
| $$m∠D=90˚, m∠E=90˚ $$$$DC=7, EF=7, $$$$BD=24, GE=24$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$GL=5x, KL=4x+7, $$$$x=7$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$90˚=90˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$GL=5(7)$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠D=m∠E$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$GL=35$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠D≅∠E$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$KL=4\left(7\right)+7$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$7=7$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$KL=35$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$DC=EF$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$GL=KL$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$\overbar{DC}≅\overbar{EF}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{GL}≅\overbar{KL}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$24=24$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{LH}≅\overbar{LH}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$BD=GE$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$H is the midpoint of \overbar{GK}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$\overbar{BD}≅\overbar{GE}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{GH}≅\overbar{KH}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△BCD≅△GFE$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△GLH≅△KLH$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

|  |  |  |
| --- | --- | --- |
| 7. |  | **PROOF 7 Continued** |
| Given: $x=0, z=13, y=73$  Prove: $△GHJ≅△LKM$ |  | $$\overbar{GH}≅\overbar{LK}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$y˚=y˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠G=m∠L$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠G≅∠L$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| Statements | Reasons |  | $$m∠H=\left(2\left(13\right)-12\right)˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠G=y˚, m∠L=y˚, $$$$m∠H=\left(2z-12\right)˚, $$$$m∠K=(z+1)˚, $$$$GH=7x+3, $$$$LK=8x+3, $$$x=0, z=13, y=73$  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠H=14˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠K=\left(\left(13\right)+1\right)˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$GH=7\left(0\right)+3$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠K=14˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$GH=3$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠H=m∠K$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$LK=8\left(0\right)+3$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠H≅∠K$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$LK=3$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△GHJ≅△LKM$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$GH=LK$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  |  |  |

8.

|  |
| --- |
| Given: $\overbar{AE}≅\overbar{CD}, B is the midpoint of \overbar{DE}, B is the midpoint of \overbar{AC}$ Prove: $△ABE≅△CBD$ |
| Statements | Reasons |
|  |  |