Angle-Angle-Side Triangle Congruence (AAS)

There are 5 properties that are used to prove triangles congruent: **SSS,** **SAS,** **ASA,** **AAS,** and HL. We have already talked about SSS, SAS, & ASA. Today, we are going to discuss **AAS**. Angle-Angle-Side means that if two angles of a triangle and the side that **is not** between them are congruent to the two angles and the side that **is not** between them on another triangle, then the triangles are congruent. So, if the angle, then angle, then side are the same on the two triangles, then the triangles are the same.

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| **Angle-Angle-Side (AAS) Triangle Congruence** | **Example proof using AAS** |
| **Setup steps:** Angle1 on Triangle1 is congruent to Angle1 on Triangle2 Angle2 on Triangle1 is congruent to  Angle2 on Triangle2 Side **not** between Angles1&2 on Triangle1 is congruent to  Side **not** between Angles1&2 on Triangle2**Resulting step:** The triangles are congruent by **AAS**. |

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| Given: $∠B≅∠F, ∠C≅∠E, \overbar{CA}≅\overbar{ED}$ Prove: $△ABC≅△DFE$ |
| Statements | Reasons |
| $$∠B≅∠F$$ | Given |
| $$∠C≅∠E$$ | Given |
| $$\overbar{CA}≅\overbar{ED}$$ | Given |
| $$△ABC≅△DFE$$ | Angle-Angle-Side (AAS) |

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Fill in the reasons on the proofs below. Be careful, though, you now know 4 ways to prove that triangles are congruent.

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| 1. |  | 2. |
| Given: $∠A≅∠D \overbar{AB}≅\overbar{DF}, and ∠B≅∠F$ Prove: $△ABC≅△DFE$ |  | Given: $\overbar{CA}≅\overbar{ED}, \overbar{AB}≅\overbar{DF}, and \overbar{BC}≅\overbar{FE}$ Prove: $△ABC≅△DFE$ |
| Statements | Reasons |  | Statements | Reasons |
| $$∠A≅∠D \overbar{AB}≅\overbar{DF}, $$$$and ∠B≅∠F$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{CA}≅\overbar{ED}, \overbar{AB}≅\overbar{DF}, $$$$and \overbar{BC}≅\overbar{FE}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△ABC≅△DFE$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△ABC≅△DFE$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| 3. |  | 4. |
| Given: $∠B≅∠F, ∠C≅∠E, and \overbar{CA}≅\overbar{ED}$ Prove: $△ABC≅△DFE$ |  | Given: $\overbar{BC}≅\overbar{FE}, ∠C≅∠E, and \overbar{CA}≅\overbar{ED}$ Prove: $△ABC≅△DFE$ |
| Statements | Reasons |  | Statements | Reasons |
| $$∠B≅∠F$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{BC}≅\overbar{FE}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠C≅∠E$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠C≅∠E$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$\overbar{CA}≅\overbar{ED}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{CA}≅\overbar{ED}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△ABC≅△DFE$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△ABC≅△DFE$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

Hypotenuse-Leg Triangle Congruence (HL)

The last triangle congruence property is **HL**. Hypotenuse-Leg means that if the **side not touching the 90˚ angle** **(hypotenuse)** of a triangle and one of the **side**s that is **touching the 90˚ angle (leg)** are congruent to the **hypotenuse** and a **leg** on another triangle, then the triangles are congruent. So, if, **on a right triangle,** the side not touching the right angle (hypotenuse) and a side touching the right angle (leg) are the same on the two triangles, then the triangles are the same.

The Hypotenuse-Leg Triangle Congruence property is special, because it is the only property that **requires** **a right triangle**. That is why we no longer use the terms “angle” and “side” to describe the relationship—all triangles have sides, but only one type of triangle has a hypotenuse and legs (a right triangle).

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| **Hypotenuse-Leg (HL) Triangle Congruence** | **Example proof using HL** |
| **Setup steps:** Triangle1 and Triangle2 are right triangles Hypotenuse on RightTriangle1 is congruent to Hypotenuse on RightTriangle2 Leg on RightTriangle1 is congruent to  Leg on RightTriangle2**Resulting step:** The triangles are congruent by **HL**. |

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| Given: $∠B≅∠F, ∠C≅∠E, \overbar{CA}≅\overbar{ED}$ Prove: $△ABC≅△DFE$ |
| Statements | Reasons |
| $△ABC \& △DFE$ are right triangles | Given |
| $$\overbar{BC}≅\overbar{FE}$$ | Given |
| $$\overbar{AB}≅\overbar{DF}$$ | Given |
| $$△ABC≅△DFE$$ | Hypotenuse-Leg (HL) |

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| 5. |  | 6. |
| Given: $\overbar{AB}≅\overbar{EF}, and \overbar{BC}≅\overbar{FD}$ Prove: $△ABC≅△EFD$ |  | Given: $∠A≅∠E, \overbar{AB}≅\overbar{EF}, and \overbar{CA}≅\overbar{DE}$ Prove: $△ABC≅△EFD$ |
| Statements | Reasons |  | Statements | Reasons |
| $$\overbar{AB}≅\overbar{EF}, \overbar{BC}≅\overbar{FD}, $$$$m∠A=90˚ and $$$$m∠E=90˚$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠A≅∠E, \overbar{AB}≅\overbar{EF}, $$and $\overbar{CA}≅\overbar{DE}$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△ABC≅△EFD$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△ABC≅△EFD$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| 7. |  | 8. |
| Given:  Prove: $△LNM≅△PNM$ |  | Given: $m∠LNM=m∠PMN$  Prove: $△LNM≅△PMN$ |
| Statements | Reasons |  | Statements | Reasons |
| $$\overbar{LM}≅\overbar{PM}, $$$$m∠LNM=90˚ \& $$$m∠PNM=90˚$  | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠L≅∠P$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$m∠LNM=m∠PNM$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$m∠LNM=m∠PMN$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$∠LNM≅∠PNM$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$∠LNM≅∠PMN$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$\overbar{MN}≅\overbar{MN}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$\overbar{MN}≅\overbar{NM}$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| $$△LNM≅△PNM$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |  | $$△LNM≅△PMN$$ | \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 9. |  | 10. |
|  Given: $AB=5, LM=5, m∠B=90˚, m∠M=90˚$  and $\overbar{BC}≅\overbar{MN}$ Prove: $△ABC≅△LMN$ |  |  Given: $AC=3x, LN=2x+4, AB=x, LM=2x-4,$ $x=4, m∠M=90˚$ $and$ $m∠B=90˚$Prove: $△ABC≅△LMN$ |
| Statements | Reasons |  | Statements | Reasons |
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| 11. |  | 12. |
|  Given: $AB=8, BC=15, AC=5x+2, LN=17, $ $MN=15, LM=8 $ $and x=3$Prove: $△ABC≅△LMN$ |  |  Given: $m∠B=90˚, m∠M=90˚, ∠A≅∠L, $ $AC=9x, and $ $LN=9x$Prove: $△ABC≅△LMN$ |
| Statements | Reasons |  | Statements | Reasons |
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