

## Determining if Lines are Parallel

For two-column proofs about **lines cut by a transversal**, you can use parallel lines to **prove angle relationships** or angle relationships to **prove lines are parallel** (always uses **converse**).

There are eight proof properties

Proving Angle Relationships (end with angles)	Proving Lines Parallel (end with parallel lines)												
<p><b>Corresponding Angles Postulate:</b> If the lines are parallel, then corresponding angles are congruent.  <b>EXAMPLE:</b> <math>\angle 6</math> and <math>\angle 7</math> are corresponding angles on lines <math>a</math> and <math>b</math>. <math>a \parallel b</math>.</p> <table border="1"> <thead> <tr> <th>Statements</th> <th>Reasons</th> </tr> </thead> <tbody> <tr> <td>1. <math>a \parallel b</math></td> <td>1. Given</td> </tr> <tr> <td>2. <math>\angle 6 \cong \angle 7</math></td> <td>2. <b>Corresponding Angles Postulate</b></td> </tr> </tbody> </table>	Statements	Reasons	1. $a \parallel b$	1. Given	2. $\angle 6 \cong \angle 7$	2. <b>Corresponding Angles Postulate</b>	<p><b>Converse of the Corresponding Angles Postulate:</b> If corresponding angles are congruent, then the lines are parallel.  <b>EXAMPLE:</b> <math>\angle 6</math> and <math>\angle 7</math> are corresponding angles on lines <math>a</math> and <math>b</math>. <math>\angle 6 \cong \angle 7</math>.</p> <table border="1"> <thead> <tr> <th>Statements</th> <th>Reasons</th> </tr> </thead> <tbody> <tr> <td>1. <math>\angle 6 \cong \angle 7</math></td> <td>1. Given</td> </tr> <tr> <td>2. <math>a \parallel b</math></td> <td>2. <b>Converse of the Corresponding Angles Postulate</b></td> </tr> </tbody> </table>	Statements	Reasons	1. $\angle 6 \cong \angle 7$	1. Given	2. $a \parallel b$	2. <b>Converse of the Corresponding Angles Postulate</b>
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Fill in the missing parts to each two-column proof.

1.  $\angle 1$  and  $\angle 2$  are alternate interior angles on lines  $a$  &  $b$ .  
 $a \parallel b$ .

Statements	Reasons
1. $a \parallel b$	1. Given
2. $\angle 1 \cong \angle 2$	2. _____

2.  $\angle 9$  and  $\angle 10$  are alternate exterior angles on lines  $m$  &  $n$ .  
 $\angle 9 \cong \angle 10$ .

Statements	Reasons
1. $\angle 9 \cong \angle 10$	1. Given
2. $m \parallel n$	2. _____

3.  $\angle 2$  and  $\angle 6$  are corresponding angles on lines  $x$  &  $y$ .  
 $x \parallel y$ .

Statements	Reasons
1. $x \parallel y$	1. Given
2. $\angle 2 \cong \angle 6$	2. _____

4.  $\angle 2$  and  $\angle 4$  are same side interior angles on lines  $q$  &  $t$ .  
 $m\angle 2 + m\angle 4 = 180^\circ$ .

Statements	Reasons
1. $m\angle 2 + m\angle 4 = 180^\circ$	1. Given
2. $q \parallel t$	2. _____

5.  $\angle 3$  and  $\angle 4$  are corresponding angles on lines  $r$  &  $s$ .  
 $\angle 3 \cong \angle 4$ .

Statements	Reasons
1. $\angle 3 \cong \angle 4$	1. Given
2. $r \parallel s$	2. _____

6.  $\angle 5$  and  $\angle 6$  are same side interior angles on lines  $c$  &  $d$ .  
 $c \parallel d$ .

Statements	Reasons
1. $c \parallel d$	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$	2. _____

7.  $\angle 6$  and  $\angle 8$  are alternate exterior angles on lines  $f$  &  $g$ .  
 $f \parallel g$ .

Statements	Reasons
1. $f \parallel g$	1. Given
2. $\angle 6 \cong \angle 8$	2. _____

8.  $\angle 11$  and  $\angle 12$  are alternate interior angles on lines  $y$  &  $z$ .  
 $\angle 11 \cong \angle 12$ .

Statements	Reasons
1. $\angle 11 \cong \angle 12$	1. Given
2. $y \parallel z$	2. _____

Now, try your hand at more complicated proofs.

9.  
 Given:  $\angle 3$  and  $\angle 4$  are same side interior angles on lines  $a$  &  $b$ .  $m\angle 3 = 60^\circ$ .  $m\angle 4 = 120^\circ$ .  
 Prove:  $a \parallel b$

Statements	Reasons
1. $m\angle 3 = 60^\circ$ $m\angle 4 = 120^\circ$	1. _____
2. $m\angle 3 + m\angle 4 = 60 + 120$	2. _____ <i>Hint: I plugged in stuff</i>
3. $m\angle 3 + m\angle 4 = 180^\circ$	3. _____
4. $a \parallel b$	4. _____

10.  
 Given:  $\angle 1$  and  $\angle 2$  are corresponding angles on lines  $c$  &  $d$ .  $c \parallel d$ .  $m\angle 1 = 5x^\circ$ .  $m\angle 2 = (4x + 20)^\circ$ .  
 Prove:  $m\angle 1 = 100^\circ$

Statements	Reasons
1. $m\angle 1 = 5x^\circ$ $m\angle 2 = (4x + 20)^\circ$	1. _____
2. $c \parallel d$	2. _____
3. $\angle 1 \cong \angle 2$	3. _____
4. $m\angle 1 = m\angle 2$	4. _____
5. $5x = 4x + 20$	5. _____
6. $x = 20$	6. _____
7. $m\angle 1 = 5(20)$	7. _____
8. $m\angle 1 = 100^\circ$	8. _____