Determining if Lines are Parallel

For two-column proofs about **lines cut by a transversal**, you can use parallel lines to **prove angle relationships** orangle relationships to **prove lines are parallel** (always uses **converse**).

There are eight proof properties

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| **Proving Angle Relationships (end with angles)** | **Proving Lines Parallel (end with parallel lines)** |
| **Corresponding Angles Postulate:** If the lines are parallel, then corresponding angles are congruent.EXAMPLE: $∠6$ and $∠7$ are corresponding angles on lines *a* and *b*. *a* || *b*.

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| Statements | Reasons |
| 1. *a* || *b* | 1. Given |
| 2. $∠6≅∠7$ | 2. **Corresponding Angles Postulate** |

. | **Converse of the Corresponding Angles Postulate:** If corresponding angles are congruent, then the lines are parallel.EXAMPLE: $∠6$ and $∠7$ are corresponding angles on lines *a* and *b*. $∠6≅∠7$.

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| Statements | Reasons |
| 1. $∠6≅∠7$ | 1. Given |
| 2. *a* || *b* | 2.  **Converse of the** **Corresponding Angles Postulate** |

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| **Alternate Interior Angles Theorem:** If the lines are parallel, then alternate interior angles are congruent.EXAMPLE: $∠2$ and $∠5$ are alternate interior angles on lines *m* and *n*. *m* || *n*.

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| Statements | Reasons |
| 1. *m* || *n* | 1. Given |
| 2. $∠2≅∠5$ | 2. **Alternate Interior Angles Theorem** |

. | **Converse of the Alternate Interior Angles Theorem:** If alternate interior angles are congruent, then the lines are parallel.EXAMPLE: $∠2$ and $∠5$ are alternate interior angles on lines *m* and *n*. $∠2≅∠5$.

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| Statements | Reasons |
| 1. $∠2≅∠5$ | 1. Given |
| 2. *m* || *n* | 2.  **Converse of the** **Alternate Interior Angles Theorem** |

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| **Alternate Exterior Angles Theorem:** If the lines are parallel, then alternate exterior angles are congruent.EXAMPLE: $∠1$ and $∠3$ are alternate exterior angles on lines *p* and *q*. *p* || *q*.

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| Statements | Reasons |
| 1. *p* || *q* | 1. Given |
| 2. $∠1≅∠3$ | 2. **Alternate Interior Angles Theorem** |

. | **Converse of the Alternate exterior Angles Theorem:** If alternate exterior angles are congruent, then the lines are parallel.EXAMPLE: $∠1$ and $∠3$ are alternate exterior angles on lines *a* and *b*. $∠1≅∠3$.

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| Statements | Reasons |
| 1. $∠1≅∠3$ | 1. Given |
| 2. *p* || *q* | 2.  **Converse of the** **Alternate Interior Angles Theorem** |

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| **Same Side Interior Angles Theorem:** If the lines are parallel, then same side interior angles add to equal 180˚.EXAMPLE: $∠8$ and $∠9$ are same side interior angles on lines *t* and *v*. *t* || *v*.

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| Statements | Reasons |
| 1. *t* || *v* | 1. Given |
| 2. $m∠8+m∠9=180˚$ | 2. **Same Side Interior Angles Theorem** |

. | **Converse of the Same Side Interior Angles Theorem:** If same side interior angles add to equal 180˚, then the lines are parallel.EXAMPLE: $∠8$ and $∠9$ are same side interior angles on lines *t* and *v*. $m∠8+m∠9=180˚$.

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| Statements | Reasons |
| 1. $m∠8+m∠9=180˚$ | 1. Given |
| 2. *t* || *v* | 2.  **Converse of the** **Same Side Interior Angles Theorem** |

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Fill in the missing parts to each two-column proof.

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| 1. $∠1$ and $∠2$ are alternate interior angles on lines *a* & *b*. *a* || *b*.

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| Statements | Reasons |
| 1. *a* || *b* | 1. Given |
| 2. $∠1≅∠2$  | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

. | 5. $∠3$ and $∠4$ are corresponding angles on lines *r* & *s*. $∠3≅∠4$.

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| Statements | Reasons |
| 1. $∠3≅∠4$ | 1. Given |
| 2. *r* || *s* | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| 2. $∠9$ and $∠10$ are alternate exterior angles on lines *m* & *n*. $∠9≅∠10$.

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| Statements | Reasons |
| 1. $∠9≅∠10$ | 1. Given |
| 2. *m* || *n* | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

 | 6. $∠5$ and $∠6$ are same side interior angles on lines *c* & *d*. *c* || *d*.

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| Statements | Reasons |
| 1. *c* || *d* | 1. Given |
| 2. $m∠1+m∠2=180˚$  | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| 3. $∠2$ and $∠6$ are corresponding angles on lines *x* & *y.**x* || *y*.

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| Statements | Reasons |
| 1. *x* || *y* | 1. Given |
| 2. $∠2≅∠6$ | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

 | 7. $∠6 $and $∠8$ are alternate exterior angles on lines *f* & *g*. *f* || *g*.

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| Statements | Reasons |
| 1. *f* || *g* | 1. Given |
| 2. $∠6≅∠8$ | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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| 4. $∠2$ and $∠4$ are same side interior angles on lines *q* & *t*. $m∠2+m∠4=180˚$.

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| Statements | Reasons |
| 1. $ m∠2+m∠4=180˚$ | 1. Given |
| 2. *q* || *t* | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

. | 8. $∠11$ and $∠12$ are alternate interior angles on lines *y* & *z*. $∠11≅∠12$.

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| Statements | Reasons |
| 1. $∠11≅∠12$ | 1. Given |
| 2. *y* || *z* | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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Now, try your hand at more complicated proofs.

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| 9. Given: $∠3 $and $∠4$ are same side interior angles on lines *a* & *b.* $m∠3=60˚$*.* $m∠4=120˚$*.*Prove: *a* || *b*

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| Statements | Reasons |
| 1. $ m∠3=60˚$$$m∠4=120˚$$$∠3 $and $∠4$ are S.S. Int. $∠s$ | 1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 2. $$m∠3+m∠4=m∠3+m∠4$$ | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 3.$ m∠3+m∠4=60+120$ | 3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 4.$ m∠3+m∠4=180˚$ | 4. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 5. *a* || *b* | 5. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

 | 10. Given: $∠1 $and $∠2$ are corresponding angles on lines *c* & *d. c* || *d.* $m∠1=5x˚$*.* $m∠2=(4x+20)˚$*.*Prove: $m∠1=100˚$

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| Statements | Reasons |
| 1. $ m∠1=5x˚$$m∠2=(4x+20)˚$ | 1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 2. *c* || *d*$∠1 $& $∠2$ are corr. $∠s$ | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 3.$ ∠1≅∠2$ | 3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 4. $m∠1=m∠2$ | 4. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 5. 5*x* = 4*x* + 20 | 5. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 6. *x* = 20 | 6. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 7. $m∠1=5(20)$ | 7. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |
| 8. $m∠1=100˚$ | 8. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |

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