Determining if Lines are Parallel

For two-column proofs about **lines cut by a transversal**, you can use parallel lines to **prove angle relationships** orangle relationships to **prove lines are parallel** (always uses **converse**).

There are eight proof properties

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| **Proving Angle Relationships (end with angles)** | **Proving Lines Parallel (end with parallel lines)** |
| **Corresponding Angles Postulate:** If the lines are parallel, then corresponding angles are congruent.  EXAMPLE: and are corresponding angles on lines *a* and *b*. *a* || *b*.   |  |  | | --- | --- | | Statements | Reasons | | 1. *a* || *b* | 1. Given | | 2. | 2. **Corresponding Angles Postulate** |   . | **Converse of the Corresponding Angles Postulate:** If corresponding angles are congruent, then the lines are parallel.  EXAMPLE: and are corresponding angles on lines *a* and *b*. .   |  |  | | --- | --- | | Statements | Reasons | | 1. | 1. Given | | 2. *a* || *b* | 2.  **Converse of the** **Corresponding Angles Postulate** |   . |
| **Alternate Interior Angles Theorem:** If the lines are parallel, then alternate interior angles are congruent.  EXAMPLE: and are alternate interior angles on lines *m* and *n*. *m* || *n*.   |  |  | | --- | --- | | Statements | Reasons | | 1. *m* || *n* | 1. Given | | 2. | 2. **Alternate Interior Angles Theorem** |   . | **Converse of the Alternate Interior Angles Theorem:** If alternate interior angles are congruent, then the lines are parallel.  EXAMPLE: and are alternate interior angles on lines *m* and *n*. .   |  |  | | --- | --- | | Statements | Reasons | | 1. | 1. Given | | 2. *m* || *n* | 2.  **Converse of the** **Alternate Interior Angles Theorem** |   . |
| **Alternate Exterior Angles Theorem:** If the lines are parallel, then alternate exterior angles are congruent.  EXAMPLE: and are alternate exterior angles on lines *p* and *q*. *p* || *q*.   |  |  | | --- | --- | | Statements | Reasons | | 1. *p* || *q* | 1. Given | | 2. | 2. **Alternate Interior Angles Theorem** |   . | **Converse of the Alternate exterior Angles Theorem:** If alternate exterior angles are congruent, then the lines are parallel.  EXAMPLE: and are alternate exterior angles on lines *a* and *b*. .   |  |  | | --- | --- | | Statements | Reasons | | 1. | 1. Given | | 2. *p* || *q* | 2.  **Converse of the** **Alternate Interior Angles Theorem** |   . |
| **Same Side Interior Angles Theorem:** If the lines are parallel, then same side interior angles add to equal 180˚.  EXAMPLE: and are same side interior angles on lines *t* and *v*. *t* || *v*.   |  |  | | --- | --- | | Statements | Reasons | | 1. *t* || *v* | 1. Given | | 2. | 2. **Same Side Interior Angles Theorem** |   . | **Converse of the Same Side Interior Angles Theorem:** If same side interior angles add to equal 180˚, then the lines are parallel.  EXAMPLE: and are same side interior angles on lines *t* and *v*. .   |  |  | | --- | --- | | Statements | Reasons | | 1. | 1. Given | | 2. *t* || *v* | 2.  **Converse of the** **Same Side Interior Angles Theorem** |   . |

Fill in the missing parts to each two-column proof.

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| 1. and are alternate interior angles on lines *a* & *b*. *a* || *b*.   |  |  | | --- | --- | | Statements | Reasons | | 1. *a* || *b* | 1. Given | | 2. | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |   . | 5. and are corresponding angles on lines *r* & *s*. .   |  |  | | --- | --- | | Statements | Reasons | | 1. | 1. Given | | 2. *r* || *s* | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | |
| 2. and are alternate exterior angles on lines *m* & *n*. .   |  |  | | --- | --- | | Statements | Reasons | | 1. | 1. Given | | 2. *m* || *n* | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 6. and are same side interior angles on lines *c* & *d*. *c* || *d*.   |  |  | | --- | --- | | Statements | Reasons | | 1. *c* || *d* | 1. Given | | 2. | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |   . |
| 3. and are corresponding angles on lines *x* & *y.*  *x* || *y*.   |  |  | | --- | --- | | Statements | Reasons | | 1. *x* || *y* | 1. Given | | 2. | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 7. and are alternate exterior angles on lines *f* & *g*.  *f* || *g*.   |  |  | | --- | --- | | Statements | Reasons | | 1. *f* || *g* | 1. Given | | 2. | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |   . |
| 4. and are same side interior angles on lines *q* & *t*. .   |  |  | | --- | --- | | Statements | Reasons | | 1. | 1. Given | | 2. *q* || *t* | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |   . | 8. and are alternate interior angles on  lines *y* & *z*. .   |  |  | | --- | --- | | Statements | Reasons | | 1. | 1. Given | | 2. *y* || *z* | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ |   . |
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Now, try your hand at more complicated proofs.

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| 9.  Given: and are same side interior angles on  lines *a* & *b. . .*  Prove: *a* || *b*   |  |  | | --- | --- | | Statements | Reasons | | 1.  and are S.S. Int. | 1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 2. | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 3. | 3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 4. | 4. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 5. *a* || *b* | 5. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 10.  Given: and are corresponding angles on  lines *c* & *d. c* || *d. . .*  Prove:   |  |  | | --- | --- | | Statements | Reasons | | 1. | 1. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 2. *c* || *d*  & are corr. | 2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 3. | 3. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 4. | 4. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 5. 5*x* = 4*x* + 20 | 5. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 6. *x* = 20 | 6. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 7. | 7. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | | 8. | 8. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ | |