

Determining Measures Using Congruence

When it comes to **congruent** triangles, there is a simple fact that is often overlooked—if the triangles are the same, then all of their parts must be the same as well. Basically, **Corresponding Parts of Congruent Triangles are Congruent**, or **CPCTC** for short. What this means is that, if you know triangles are congruent, then you can solve for missing angle and side measures, because you know that they are equal.

First, though, you have to be able to identify which parts of the congruent triangles that *correspond* (go together). Never trust the picture. Always use the information you're given. In this case, that information will look like this:

$$\triangle ABC \cong \triangle LMN$$

It tells you not only that the triangles are congruent, but also which parts are congruent to each other:

$\triangle ABC \cong \triangle LMN$ 	$\triangle ABC \cong \triangle LMN$ 	$\triangle ABC \cong \triangle LMN$ 	Since A is L , B is M , and C is N , $\angle A \cong \angle L$, $\angle B \cong \angle M$, and $\angle C \cong \angle N$ Also, $\overline{AB} \cong \overline{LM}$, $\overline{BC} \cong \overline{MN}$, and $\overline{AC} \cong \overline{LN}$
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Now, let's apply that information to solving triangles.

<p>EXAMPLE $\triangle DEF \cong \triangle RQP$. $DE = ?$ $EF = ?$ $DF = ?$</p> <p>If $\triangle DEF \cong \triangle RQP$, then D is R, E is Q, and F is P. So...</p> $DE = RQ = \boxed{4}$ $EF = QP = \boxed{3}$ $DF = RP = \boxed{5}$	<p>1. $\triangle GHI \cong \triangle UST$. $US = ?$ $ST = ?$ $UT = ?$</p>	<p>2. $\triangle KLN \cong \triangle CBA$. $CB = ?$ $BA = ?$ $KN = ?$</p>
<p>EXAMPLE $\triangle DEF \cong \triangle RQP$. $m\angle E = ?$ $m\angle R = ?$ $m\angle Q = ?$ $m\angle P = ?$</p> <p>$m\angle D + m\angle E + m\angle F = 180^\circ$ $40 + m\angle E + 52 = 180^\circ$ $92 + m\angle E = 180^\circ$ $m\angle E = \boxed{88^\circ}$</p> <p>If $\triangle DEF \cong \triangle RQP$, then D is R, E is Q, and F is P. So...</p> $m\angle R = m\angle D = \boxed{40^\circ}$ $m\angle Q = m\angle E = \boxed{88^\circ}$ $m\angle P = m\angle F = \boxed{52^\circ}$	<p>3. $\triangle GHI \cong \triangle UST$. $m\angle I = ?$ $m\angle U = ?$ $m\angle S = ?$ $m\angle T = ?$</p>	<p>4. $\triangle KLN \cong \triangle CBA$. $m\angle K = ?$ $m\angle N = ?$ $m\angle B = ?$ $m\angle A = ?$</p>
<p>EXAMPLE $\triangle DEF \cong \triangle PQR$. Determine the value of x.</p> <p>If $\triangle DEF \cong \triangle PQR$, then D is P, E is Q, and F is R. So...</p> $DF = PR$ <i>EF is extra info— don't use it!</i> $x - 4 = 2x - 10$ $-4 = x - 10$ $6 = x$ $x = \boxed{6}$	<p>5. $\triangle ABC \cong \triangle EFD$. Determine the value of x.</p>	<p>6. $\triangle GHI \cong \triangle LJK$. Determine the value of x.</p>

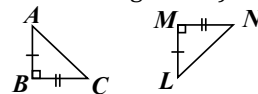
<p>EXAMPLE $\triangle ABC \cong \triangle LMN$. $AB = 16$ & $AC = 12$. $LN = ?$</p> <p><i>You don't need the pictures to know what matches. A is L, B is M, & C is N. So...$LN = AC$ (You don't need AB)</i> $LN = \boxed{12}$</p>	<p>7. $\triangle PQR \cong \triangle FED$. $ED = 7$ & $PR = 4$. $FD = ?$</p>	<p>8. $\triangle KLN \cong \triangle RST$. $RT = 19$ & $ST = 5$. $LN = ?$</p>
<p>EXAMPLE $\triangle CAT \cong \triangle DOG$. $m\angle A = (4x - 3)^\circ$, $m\angle T = (5x + 10)^\circ$, and $m\angle G = (6x - 1)^\circ$. $m\angle A = ?$</p> <p><i>C is D, A is O, & T is G. I can only solve using the ones that match. So...</i> $m\angle T = m\angle G$ $5x + 10 = 6x - 1$ $10 = x - 1$ $11 = x$ $m\angle A = 4(11) - 3 = 44 - 3$ $m\angle A = \boxed{41^\circ}$</p>	<p>9. $\triangle HER \cong \triangle MIT$. $m\angle H = (2x - 9)^\circ$, $m\angle E = (3x - 2)^\circ$, and $m\angle I = (x + 26)^\circ$. $m\angle E = ?$</p>	<p>10. $\triangle BRO \cong \triangle WNS$. $m\angle R = (x + 7)^\circ$, $m\angle N = (5x - 73)^\circ$, and $m\angle S = (x + 13)^\circ$. $m\angle S = ?$</p>
<p>EXAMPLE $\triangle DAZ \cong \triangle ELZ$. $DE = 5x + 1$, $AZ = 2x - 3$, and $LZ = x + 4$. $DZ = ?$</p> <p><i>They share a letter, so the triangles are connected. But that doesn't change the process. D is E, A is L, & Z is Z. The two I have that match are...</i> $AZ = LZ$ $2x - 3 = x + 4$ $x - 3 = 4$ $x = 7$ $DZ = 5(7) + 1 = 35 + 1 = \boxed{36}$</p>	<p>11. $\triangle LOG \cong \triangle REG$. $LG = 7x - 5$, $RG = 5x + 3$, and $EG = 4x$. $EG = ?$</p>	<p>12. $\triangle BAT \cong \triangle MAN$. $BA = 3x + 4$, $AN = 4x - 3$, and $AT = 5x - 5$. $AN = ?$</p>

And, now, for a proof... Your answer choices are:

SSS, SAS, ASA, AAS, HL, and CPCTC (the parts are the same because the triangles are).

Given: $\angle B \cong \angle M$, $\overline{AB} \cong \overline{LM}$, & $\overline{BC} \cong \overline{MN}$

Prove: $LN = AC$



Statements	Reasons
1. $\angle B \cong \angle M$, $\overline{AB} \cong \overline{LM}$, & $\overline{BC} \cong \overline{MN}$	1. Given
2. $\triangle ABC \cong \triangle LMN$	2. [?]
3. $\overline{AC} \cong \overline{LN}$	3. [?]
4. $AC = LN$	4. Definition of Congruence
5. $LN = AC$	5. Symmetric Property of Equality