

Interior Angle Sum Theorem

The **Interior Angle Sum Theorem** tells us that the total number of degrees in any given polygon depends on the number of triangles inside of it. Since each triangle has 180° , if you multiply the number of triangles by 180° you will have determined the Interior Angle Sum. Remember, to determine the number of triangles, just subtract the number of sides by two.

The Interior Angle Sum Theorem: $(n - 2)(180) = \text{Total degree measure of any polygon}$,
 where n is the number of sides.

In order to determine the number of sides, you also need to know the names of polygons...

| | | | |
|---------------|---------|-----------|----------|
| Triangle | 3 sides | Octagon | 8 sides |
| Quadrilateral | 4 sides | Nonagon | 9 sides |
| Pentagon | 5 sides | Decagon | 10 sides |
| Hexagon | 6 sides | Undecagon | 11 sides |
| Heptagon | 7 sides | Dodecagon | 12 sides |

| | |
|----------|------------------------|
| n -gon | Polygon with n sides |
|----------|------------------------|

Determine the **Interior Angle Sum** of the polygon.

| | | |
|--|---|---|
| EXAMPLE Pentagon $(n - 2)(180) = (5 - 2)(180)$ $= 3(180)$ $= \boxed{540^\circ}$ | EXAMPLE 19-gon $(n - 2)(180) = (19 - 2)(180)$ $= 17(180)$ $= \boxed{2060^\circ}$ | EXAMPLE 22-gon $(n - 2)(180) = (22 - 2)(180)$ $= 20(180)$ $= \boxed{3600^\circ}$ |
| 1. Decagon | 2. Hexagon | 3. 14-gon |
| 4. Nonagon | 5. Quadrilateral | 6. Dodecagon |
| 7. 17-gon | 8. Octagon | 9. 20-gon |

The **Exterior Angle Sum** is a bit simpler. No matter how many sides a polygon has...
 the **Exterior Angle Sum** is always 360° .

Determine the exterior angle sum.

| | | |
|---|---|---|
| EXAMPLE Pentagon $\boxed{360^\circ}$ | EXAMPLE 19-gon $\boxed{360^\circ}$ | EXAMPLE 22-gon $\boxed{360^\circ}$ |
| 10. Decagon | 11. Hexagon | 12. 14-gon |
| 13. Nonagon | 14. Quadrilateral | 15. Dodecagon |
| 16. 17-gon | 17. Octagon | 18. 20-gon |

To determine the measure of any angle of a polygon, you use the interior angle sum. Just like you would add the angles of a triangle to equal 180° , you add all of the angles of the polygon to equal the **interior angle sum**.

EXAMPLE

Determine the measure of each interior angle of a pentagon with angles $(2x - 6)$, $(x + 14)$, $(3x + 32)$, $(7x + 9)$, and $(7x + 11)$.

Interior Angle Sum of a pentagon (5-sides): $(n - 2)(180) = (5 - 2)(180) = 3(180) = 540^\circ$

Add the angles and set them equal to 540° .

$$\begin{aligned}(2x - 6) + (x + 14) + (3x + 32) + (7x + 9) + (7x + 11) &= 540^\circ \\ 20x + 60 &= 540 \\ 20x &= 480 \\ x &= 24\end{aligned}$$

$(7x + 9)$, and $(7x + 11)$

$$\begin{aligned}2x - 6 &= 2(24) - 6 = 48 - 6 = \boxed{42^\circ} & x + 14 &= (24) + 14 = \boxed{38^\circ} & 3x + 32 &= 3(24) + 32 = 72 + 32 = \boxed{104^\circ} \\ 7x + 9 &= 7(24) + 9 = 168 + 9 = \boxed{177^\circ} & 7x + 11 &= 7(24) + 11 = 168 + 11 = \boxed{179^\circ}\end{aligned}$$

19. Determine the measure of each interior angle of a pentagon with angles $(9x + 3)$, $(2x + 12)$, $(5x + 63)$, $(3x + 16)$, and $(6x + 21)$.

20. Determine the measure of each interior angle of a quadrilateral with angles $(x + 23)$, $(2x - 16)$, $(3x + 15)$, and $(4x + 8)$.

21. Determine the measure of each interior angle of a hexagon with angles $(3x - 13)$, $(4x + 16)$, $(2x + 7)$, $(x + 52)$, $(5x + 8)$, and $(5x + 10)$.

22. Determine the measure of each interior angle of a hexagon with angles $(3x + 28)$, $(6x - 7)$, $(4x + 42)$, and $(2x + 27)$.