Mathematical Operations: Division

**Dividing Numbers**

Rule #1: If you are dividing two numbers that have the same sign (+$ ÷ $+ or $-÷-$), then the result (called the “quotient”) will be positive.

Rule #2: If you are dividing two numbers that have different signs (+$ ÷-$ or $-÷$ +), then the result will be negative.

Rule #3: If you are dividing two numbers, you can write the problem as fraction, which can be simplified.

Rule #4: If you are simplifying a fraction, then you must divide both top and bottom (“numerator” and “denominator”) by the same value.

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| **Example 1** | **Example 2** | **Example 3** | **Example 4** |
| Simplify. $18÷6$*(as a fraction: 18/6)**I know that 18*$÷$*6=3, but, if I didn’t, I could reduce a fraction to figure it out:*$$\frac{18}{6}=\frac{18÷2}{6÷2}=\frac{9}{3}$$$$=\frac{9÷3}{3÷3}=\frac{3}{1}$$$$=3$$ | Simplify. $-4÷-20$*(as a fraction: -4/-20)**Since the smaller number is on top of the fraction (before the* $÷$*), I know that the answer’s absolute value will not be bigger than 1:*$$\frac{-4}{-20}=\frac{-4÷-4}{-20÷-4}=\frac{1}{5}$$ | Simplify. $-12÷3$*(as a fraction: -12/3)*$$\frac{-12}{3}=\frac{-12÷3}{3÷3}=\frac{-4}{1}$$$$=-\frac{4}{1}=-4$$ | Simplify. $7÷-14$*(as a fraction: 7/-14)**Since the smaller number is on top of the fraction (before the* $÷$*), I know that the answer’s absolute value will not be bigger than 1:*$$\frac{7}{-14}=\frac{7÷7}{-14÷7}=\frac{1}{-2}$$$$=-\frac{1}{2}$$ |
| 1. Simplify. $-33÷3$ | 2. Simplify. $-16÷-32$ | 3. Simplify. $28÷7$ | 4. Simplify. $9÷-36$ |
| 5. Simplify. $18÷2$ | 6. Simplify. $26÷-13$ | 7. Simplify. $-5÷35$ | 8. Simplify. $-4÷12$ |

Rule #5: If you are dividing a variable term by a number, then the numbers divide while the variable stays the same.

Rule #6: If you are dividing a variable term by a variable that doesn’t match (x with y, a with b…), then the variables stay the same without combining, while the numbers in front (called “coefficients”) divide.

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| **Example 5** | **Example 6** | **Example 7** | **Example 8** |
| $$Simplify. \frac{15r^{5}}{-6}$$*(15 can’t be divided by 6, but both of them can be divided by 3, so…)*$$\frac{15r^{5}}{-6}=\frac{\left(15÷3\right)r^{5}}{-6÷3}=\frac{5r^{5}}{-2}$$$$=-\frac{5r^{5}}{2}$$ | $$Simplify. \frac{11}{33a^{3}b}$$$$\frac{11}{33a^{3}b}=\frac{11÷11}{\left(33÷11\right)\left(a^{3}b\right)}$$$$=\frac{1}{\left(3\right)\left(a^{3}b\right)}=\frac{1}{3a^{3}b}$$ | $$Simplify. \frac{-2h^{7}}{-12k^{4}}$$$$\frac{-2h^{7}}{-12k^{4}}=\frac{\left(-2÷-2\right)h^{7}}{\left(-12÷-2\right)k^{4}}$$$$=\frac{1h^{7}}{6k^{4}}=\frac{h^{7}}{6k^{4}}$$ | $$Simplify. \frac{-60w^{6}}{15v}$$$$\frac{-60w^{6}}{15v}=\frac{\left(-60÷5\right)w^{6}}{\left(15÷5\right)v}$$$$=\frac{-12w^{6}}{3v}=\frac{\left(-12÷3\right)w^{6}}{\left(3÷3\right)v}$$$$=\frac{-4w^{6}}{1v}=-\frac{4w^{6}}{v}$$ |
| $$9. Simplify.\frac{-3p^{5}}{15}$$ | $$10. Simplify. \frac{-35rs^{8}}{-7}$$ | $$11. Simplify. \frac{6d^{9}}{-3f^{2}}$$ | $$12. Simplify. \frac{10y}{15z^{7}}$$ |

Rule #7: If you are dividing a variable term by a matching variable, then the exponents will subtract from each other ($top-bottom$) without changing the coefficient(s) in front.

Rule #8: If you subtract the variable exponents and the result is positive, then that variable belongs on top of the fraction, and the exponent stays positive.

Rule #9: If you subtract the variable exponents and the result is negative, then that variable belongs on the bottom of the fraction, but the exponent changes from negative to positive.

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| **Example 9** | **Example 10** | **Example 11** | **Example 12** |
| $$Simplify. \frac{-5x^{7}}{x^{3}}$$*(same as* $\frac{-5x^{7}}{1x^{3}}$*)*$$\frac{-5x^{7}}{1x^{3}}=\frac{-5}{1}\left(x^{7-3}\right)$$$$=-\frac{5}{1}\left(x^{+4}\right)$$*(exponent was positive,* *so x4 goes on top)*$$=-\frac{5x^{4}}{1}=-5x^{4}$$ | $$Simplify. \frac{6r^{2}}{2r^{9}}$$$$\frac{6r^{2}}{2r^{9}}=\frac{6÷2}{2÷2}\left(r^{2-9}\right)$$$$=\frac{3}{1}(r^{-7})$$ *(exponent was negative,* *so* $r^{}$ *goes on bottom)*$$=\frac{3}{1r^{+7}}=\frac{3}{r^{7}}$$ | $$Simplify. \frac{-4m^{5}}{-8m^{6}}$$$$\frac{-4m^{5}}{-8m^{6}}=\frac{-4÷-4}{-8÷-4}\left(m^{5-6}\right)$$$$=\frac{1}{2}(m^{-1})$$ *(exponent was negative,* *so* $m^{}$ *goes on bottom)*$$=\frac{1}{2m^{+1}}=\frac{1}{2m}$$ | $$Simplify. \frac{g^{4}h^{9}}{-g^{8}h^{3}}$$$$\frac{1g^{4}h^{9}}{-1g^{8}h^{3}}=\frac{1}{-1}\left(g^{4-8}\right)(h^{9-3})$$$$=-\frac{1}{1}\left(g^{-4}\right)\left(h^{6}\right)$$ *(negative exponent to bottom* $(g^{})$*,* *positive to top* $h^{6}$*)*$$=-\frac{1h^{6}}{1g^{+4}}=-\frac{h^{6}}{g^{4}}$$ |
| $$13. Simplify. \frac{w^{8}}{3w^{3}}$$ | $$14. Simplify. \frac{c^{4}}{-4c^{6}}$$ | $$15. Simplify. -\frac{14r^{6}}{7r^{2}}$$ | $$16. Simplify. \frac{4y^{7}z^{5}}{y^{2}z^{9}}$$ |

Rule #9: If you are dividing a number from a group, then you must divide the number from every term in the group.

Rule #10: If you cannot divide a number from every term in the group, then you cannot divide by that number – look for a smaller number that can divide from all involved terms.

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| **Example 13** | **Example 14** | **Example 15**$ $ | **Example 16** |
| $$Simplify. \frac{5x+20}{5}$$*(5 divides from both, so…)*$$\frac{5x+20}{5}$$$$=\frac{\left(5x÷5\right)+\left(20÷5\right)}{5÷5}$$$$=\frac{1x+4}{1}=x+4$$*Another way to think of it is as a backwards multiplication table. You know the inside, so figure out what multiplied to make it.*

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|  | *?* | *?* |
| 5 | 5x | +20 |

 | $$Simplify. \frac{4b-36}{-2}$$*(-2 divides from both, so…)*$$\frac{4b-36}{-2}$$$$=\frac{\left(4b÷-2\right)-\left(36÷-2\right)}{-2÷-2}$$$$=\frac{-2b-(-18)}{1}$$$$=-2b-(-18)$$$$=-2b+18$$*or:*

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|  | *?* | *?* |
| -2 | 4b | -36 |

 | $$Simplify. \frac{-21y-12}{-6}$$*(-6 doesn’t divide from both -21 & -12, so I need to find a number that divides from them all. -1 will, so…)*$$\frac{-21y-12}{-6}$$$$=\frac{\left(-21y÷-1\right)-\left(12÷-1\right)}{-6÷-1}$$$$=\frac{21y-(-12)}{6}$$$$=\frac{21y+12}{6}$$$$=\frac{\left(21y÷3\right)+\left(12÷3\right)}{6÷3}$$$$=\frac{7y+4}{2}$$ | $$Simplify. \frac{14r+42}{42}$$*(42 doesn’t divide from both 14r & 42, but 7 will divide from all of them, so…)*$$\frac{14r+42}{42}$$$$=\frac{\left(14r÷7\right)+\left(42÷7\right)}{42÷7}$$$$=\frac{2r+6}{6}$$$$=\frac{\left(2r÷2\right)+\left(6÷2\right)}{6÷2}$$$$=\frac{1r+3}{3}=\frac{r+3}{3}$$ |
| $$17. Simplify. \frac{6g+18}{-6}$$ | $$18. Simplify. \frac{16f-40}{8}$$ | $$19. Simplify. \frac{12x+4}{6}$$ | $$20. Simplify.\frac{-3x+15}{9}$$ |