Mathematical Operations: Multiplication

**Multiplying Numbers**

Rule #1: If you are multiplying two numbers that have the same sign (+$∙$+ or $-∙-$), then the result (called the “product”) will be positive.

Rule #2: If you are multiplying two numbers that have different signs (+$∙-$ or $-∙$ +), then the result will be negative.

Rule #3: If you are multiplying two numbers, you can write the problem as a repeated addition or a repeated subtraction problem.

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| **Example 1** | **Example 2** | **Example 3** | **Example 4** |
| Simplify. $5∙3$*(same as +5*$ ∙$ *+3)*

|  |  |
| --- | --- |
|  | +5 |
| +3 | 15 |

$+5∙+3$***is +5 added 3 times***$$=+5+5+5=15$$ ***or +3 added 5 times***$$=+3+3+3+3+3=15$$ | Simplify. $(-2)(-6)$

|  |  |
| --- | --- |
|  | -2 |
| -6 | 12 |

$ (-2)(-6)$ ***is -2 subtracted 6 times***$$=-\left(-2\right)-\left(-2\right)-(-2)$$$$-\left(-2\right)-\left(-2\right)-(-2)$$$$=+2+2+2+2+2+2$$$$=12$$***or -6 subtracted 2 times***$$=-\left(-6\right)-\left(-6\right)$$$$=+6+6=12$$ | Simplify. $4(-3)$*(same as +4*$ ∙$ *-3)*

|  |  |
| --- | --- |
|  | +4 |
| -3 | -12 |

$+4(-3)$***is +4 subtracted 3 times***$$=-\left(+4\right)-\left(+4\right)-\left(+4\right)$$$$=-4-4-4=-12$$ ***or -3 added 4 times***$$=+\left(-3\right)+\left(-3\right)+$$$$\left(-3\right)+\left(-3\right)$$$$=-3-3-3-3$$$$=-12$$ | Simplify. $-4∙1$*(same as -4*$ ∙$ *+1)*

|  |  |
| --- | --- |
|  | -4 |
| +1 | -4 |

$-4(+1)$***is -4 added 1 time***$$=+(-4)$$$$=-4$$ ***or +1 subtracted 4 times***$$=-\left(+1\right)-\left(+1\right)-$$$$\left(+1\right)-(+1)$$$$=-1-1-1-1$$$$=-4$$ |
| 1. Simplify. $-3∙-4$ | 2. Simplify. $(5)(2)$ | 3. Simplify. $-8(3)$ | 4. Simplify. $2∙(-1)$ |
| 5. Simplify. $(-6)(7)$ | 6. Simplify. $6(5)$ | 7. Simplify. $4∙-9$ | 8. Simplify. $-9∙-3$ |

Rule #3: If you are multiplying a variable term by a number, then the numbers will multiply while the variable stays the same.

Rule #4: If you are multiplying a variable term by a variable that doesn’t match (x with y, a with b…), then the variables sit next to each other without combining, but the numbers in front (called “coefficients”) multiply.

|  |  |  |  |
| --- | --- | --- | --- |
| **Example 5** | **Example 6** | **Example 7** | **Example 8** |
| Simplify. $(3x^{2})(-2)$

|  |  |
| --- | --- |
|  | $$+3x^{2}$$ |
| -2 | $-6x^{2}$  |

$$\left(3x^{2}\right)\left(-2\right)=(3)(-2)(x^{2})$$***is (-2 added 3 times)(***$x^{2}$***)***$$=-6x^{2}$$***or (+3 subtracted 2 times)(***$x^{2}$***)***$$=-6x^{2}$$ | Simplify. $(4z)(3)$

|  |  |
| --- | --- |
|  | +4z |
| +3 | 12z |

$$(4z)(3)=(4)(3)(z)$$***is (+3 added 4 times)(z)***$$=12z$$***or (+4 added 3 times)(z)***$$=12z$$ | Simplify. $(-r)(-s^{2})$*(same as* $-1r∙-1s^{2}$*)*

|  |  |
| --- | --- |
|  | -1r |
| $$-1s^{2}$$ | $$rs^{2}$$ |

$$\left(-1r\right)\left(-1s^{2}\right)$$$$=(-1)(-1)(r)(s^{2})$$***is (-1 subtracted 1 time)(***$r∙s^{2}$***)***$$=(+1)rs^{2}=rs^{2}$$***or(-1 subtracted 1 time)(***$r∙s^{2}$***)****which is the same as above* | Simplify. $(5a^{4})(2c^{3})$*(same as* $1a^{4}∙2c^{3}$*)*

|  |  |
| --- | --- |
|  | $$+5a^{4}$$ |
| $$+2c^{3}$$ | $$10a^{4}c^{3}$$ |

$$\left(5a^{4}\right)\left(2c^{3}\right)$$$$=\left(5\right)\left(2\right)\left(a^{4}\right)\left(c^{3}\right)$$***is (+5 added 2 times)(***$ a^{4}∙c^{3}$***)***$$=10a^{4}c^{3}$$***is (+2 added 5 times)(***$ a^{4}∙c^{3}$***)***$$ =10a^{4}c^{3}$$ |
| 9. Simplify. $(2x)(6)$ | 10. Simplify. $(-3)(-5g^{4})$ | 11. Simplify. $(h^{8})(-k^{3})$ | 12. Simplify. $(-4f^{2})(2g^{3})$ |
| 13. Simplify. $(-3b^{2})(2c)$ | 14. Simplify. $(6r)(7)$ | 15. Simplify. $(-2)(-6a)$ | 16. Simplify. $(4w)(-6y)$ |

Rule #5: If you are multiplying a variable term with a matching variable, then the exponents will add together without changing the coefficient in front.

|  |  |  |  |
| --- | --- | --- | --- |
| **Example 9** | **Example 10** | **Example 11** | **Example 12** |
| Simplify. $(3x^{2})(x^{4})$*(same as 3*$x^{2}∙1x^{4}$*)*

|  |  |
| --- | --- |
|  | $$3x^{2}$$ |
| $$x^{4}$$ | $$3x^{6}$$ |

$$\left(3x^{2}\right)\left(1x^{4}\right)$$$$=(3)(1)(x^{2})(x^{4})$$***to multiply the x’s, add their exponents***$$=(3)(1)(x^{2+4})=3x^{6}$$ | Simplify. $(5a^{6})(2a^{3})$

|  |  |
| --- | --- |
|  | $$5a^{6}$$ |
| $$2a^{3}$$ | $$10a^{9}$$ |

$$\left(5a^{6}\right)\left(2a^{3}\right)$$$$=(5)(2)(a^{6})(a^{3})$$***to multiply the a’s, add their exponents***$$=(5)(2)(a^{6+3})=10a^{9}$$ | Simplify. $(m^{2}p^{3})(m)$*(same as*$ 1m^{2}p^{3}∙1m^{1}$*)*

|  |  |
| --- | --- |
|  | $$1m^{2}p^{3}$$ |
| $$1m^{1}$$ | $$m^{3}p^{3}$$ |

$$\left(1m^{2}p^{3})(1m^{1}\right)$$$$=\left(1\right)\left(1\right)\left(m^{2}\right)\left(m^{1}\right)\left(p^{3}\right)$$***to multiply the m’s, add their exponents***$$=\left(1\right)\left(1\right)\left(m^{2+1}\right)\left(p^{3}\right)$$***different letters can’t combine***$$=(1)(m^{3})(p^{3})=m^{3}p^{3}$$ | Simplify. $\left(8r^{7}\right)\left(2r^{3}s^{5}\right)$

|  |  |
| --- | --- |
|  | $$2r^{3}s^{5}$$ |
| $$8r^{7}$$ | $$16r^{10}s^{5}$$ |

$$\left(8r^{7})(2r^{3}s^{5}\right)$$$$=\left(8\right)\left(2\right)\left(r^{7}\right)\left(r^{3}\right)\left(s^{5}\right)$$***to multiply the r’s, add their exponents***$$=\left(8\right)\left(2\right)\left(r^{7+3}\right)\left(s^{5}\right)$$***different letters can’t combine***$$=(16)(r^{10})(s^{5})=16r^{10}s^{5}$$ |
| 17. Simplify. $(2t^{8})(t^{2})$ | 18. Simplify. $(3s^{6})(-7s^{5})$ | 19. Simplify. $(2k^{2})(4k^{5})(t)$ | 20. Simplify. $(m^{3}n^{4})(n)$ |
| 21. Simplify. $(r^{5})(-9r^{4}s^{8})$ | 22. Simplify. $(-8p)(p^{7})$ | 23. Simplify. $(3g^{6}h)(-7g^{2})$ | 24. Simplify. $\left(x^{2}\right)(x^{3})$ |

Rule #6: If you are multiplying a number to a group, then you must multiply the number to every term in that group.

Rule #7: If you are multiplying a group to a group, then you must multiply every part of the first group by every part of the second group.

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| **Example 13**Simplify. $-8(4x-9)$

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| --- | --- | --- |
|  | $$4x$$ | $$-9$$ |
| $$-8$$ | $$-32x$$ | $$+72$$ |

*Now, add ‘em up:*$$=-32x+72$$ | **Example 14**Simplify. $5(6x+1)$

|  |  |  |
| --- | --- | --- |
|  | $$6x$$ | $$+1$$ |
| $$5$$ | $$30x$$ | $$+5$$ |

*Now, add ‘em up:*$$=30x+5$$ | **Example 15**Simplify. $(2x+3)(5x-4)$

|  |  |  |
| --- | --- | --- |
|  | $$2x^{1}$$ | $$+3$$ |
| $$5x^{1}$$ | $$10x^{1+1}$$ | $$+15x^{1}$$ |
| $$-4$$ | $$-8x^{1}$$ | $$-12$$ |

*Now, add ‘em up:*$$10x^{1+1}+15x^{1}-8x^{1}-12 $$$$10x^{2}+15x-8x-12 $$$$=10x^{2}+7x-12 $$ | **Example 16**Simplify. $(x-7)(-3x+2)$

|  |  |  |
| --- | --- | --- |
|  | $$1x^{1}$$ | $$-7$$ |
| $$-3x^{1}$$ | $$-3x^{1+1}$$ | $$+21x^{1}$$ |
| $$+2$$ | $$+2x^{1}$$ | $$-14$$ |

*Now, add ‘em up:*$$-3x^{1+1}+21x^{1}+2x^{1}-14 $$$$-3x^{2}+21x+2x-14 $$$$=-3x^{2}+23x-14$$ |
| 25. Simplify. $-6(4x-7)$ | 26. Simplify. $2(-2x+3)$ |  27. Simplify. $$(-6x+5)(-x+8)$$ | 28. Simplify. $$(2x+7)(7x-2)$$ |