Operations on Polynomials

Addition & Subtraction

To add or subtract polynomials, all you have to do is to *combine like terms*. This means that you have to add or subtract the terms that have matching variables ($x^{9} matches x^{9} not x^{8})$.

**Remember**: when you subtract a group, everything in the parentheses is being subtracted.

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| **EXAMPLE**$$\left(2n^{7}-4n^{5}+6n^{2}+1\right)+(2n^{5}-7)$$*If you re-order them by* ***degree*** *(or total exponential power), then the like terms will line up. Now, simplify!*$$2n^{7}-4n^{5}+\overline{2n^{5}}+6n^{2}+1+\overline{ -7}$$$$$$ | **EXAMPLE**$$\left(9x^{3}+4x^{2}-9\right)-\left(6x^{4}-2x^{3}+5\right)$$*Re-write it so that the negative belongs to* ***every term in the group****.*$$9x^{3}+4x^{2}-9-(6x^{4})-\left(-2x^{3})-(+5\right)$$$$9x^{3}+4x^{2}-9-6x^{4}+2x^{3}-5$$*Re-order them by* ***degree****, & simplify!*$$\overline{-6x^{4}}+9x^{3}+\overline{2x^{3}}+4x^{2}-9-\overline{5}$$$$$$ | **EXAMPLE**$$-\left(4a^{3}+2a\right)-\left(-5a^{2}+8a\right)$$$$-(4a^{3})-\left(+2a\right)-\left(-5a^{2})-(+8a\right)$$$$-4a^{3}-2a+5a^{2}-8a$$$$-4a^{3}+5a^{2}-2a-8a$$$$$$ |
| 1. $\left(9x^{6}+12x^{2}-3\right)+(5x^{6}+8)$ | 2. $\left(5p^{7}-3p^{5}-4\right)-\left(4p^{7}-2p^{5}\right)$ | 3. $\left(11b^{9}-3b+7\right)+(5b^{7}-8)$ |
| 4. $\left(10z^{2}+3z\right)-\left(-8z^{3}-2z^{2}+2\right)$ | 5. $-\left(8x^{5}+7x^{2}\right)+\left(6x^{2}+15\right)$ | 6. $-\left(-3h-6\right)-\left(2h^{2}+7h\right)$ |
| 7. $\left(7k^{12}+2k^{2}\right)-\left(7k^{12}+2k^{2}-6k\right)$ | 8. $\left(r^{9}-r^{4}+r^{2}+3\right)+(r^{5}-2r^{2})$ | 9. $-\left(7x^{5}+8x^{3}\right)-\left(3x^{3}-6\right)$ |

Multiplication

 The easiest way to multiply polynomials is to use the box method. Multiply the terms, then add ‘em up.

 **Remember**: when you multiply matching bases, the exponents *add together*.

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| **EXAMPLE**$$-5a^{3}b(-4a^{2}b+2ab-3ab^{2})$$

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| --- | --- | --- | --- |
|  | $$-4a^{2}b$$ | $$+2ab$$ | $$-3ab^{2}$$ |
| $$-5a^{3}b$$ | $$20a^{5}b^{2}$$ | $$-10a^{4}b^{2}$$ | $$+15a^{4}b^{3}$$ |

*Now, just add ‘em up!*$$$$ | **EXAMPLE**$$(2x^{3}+4x)(7x^{2}+3x-9)$$

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| --- | --- | --- | --- |
|  | $$7x^{2}$$ | $$+3x$$ | $$-9$$ |
| $$2x^{3}$$ | $$14x^{5}$$ | $$+6x^{4}$$ | $$-18x^{3}$$ |
| $$+4x$$ | $$+28x^{3}$$ | $$+12x^{2}$$ | $$-36x$$ |

$$14x^{5}+6x^{4}\overline{-18x^{3}+28x^{3}}+12x^{2}-36x$$$$$$ | **EXAMPLE**$$(5q-3)(6q^{2}-5q+7)$$

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| --- | --- | --- | --- |
|  | $$6q^{2}$$ | $$-5q$$ | $$+7$$ |
| $$5q$$ | $$30q^{3}$$ | $$-25q^{2}$$ | $$+35q$$ |
| $$-3$$ | $$-18q^{2}$$ | $$+15q$$ | $$-21$$ |

$$30q^{3}-25q^{2}+35q-18q^{2}+15q-21$$$$30q^{3}-25q^{2}-18q^{2}+35q+15q-21$$$$$$ |
| 10. $(5r^{4}+8)\left(9r^{3}+12r^{2}-3\right)$ | 11. $\left(4m^{3}-2m^{2}\right)\left(5m^{4}-3m^{3}-4\right)$ | 12. $3x^{5}\left(11x^{9}-3x+7\right)$ |
| 13. $\left(10p^{2}+3p\right)\left(-8p^{3}-2p^{2}+2\right)$ | 14. $-7gh^{3}(2g^{3}h+4g^{2}h^{2}-9gh^{3})$ | 15. $\left(-3x-6\right)\left(2x^{2}+7x\right)$ |
| 16. $-3d^{2}e^{2}\left(7d^{5}e^{3}-8de^{4}\right)$ | 17. $\left(x^{9}-x^{4}+x^{2}+3\right)(x^{5}-2x^{2})$ | 18. $\left(7x^{5}+8x^{3}\right)\left(3x^{3}-6\right)$ |

Simplifying Rational Polynomials

 First, factor out anything you can from the numerator and the denominator.

 Second, cancel out any terms that you can *(must be attached by multiplication* ***not addition*** *in order to cancel).*

 Finally, if you need to, multiply what’s left back together.

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| **EXAMPLE**$$\frac{8x-6}{4}=\frac{(2)(4x-3)}{2(2)}=$$ | **EXAMPLE**$$\frac{3x^{2}-9x}{6x^{2}}=\frac{3x(x-3)}{3x(2x)}=$$ | **EXAMPLE**$$\frac{3x^{2}+7x+2}{4x+8}$$*Factor the top. AC = (3)(2) = 6 & B = 7**Numbers that mult. to AC & add to B…*$$\frac{3x^{2}+6x+1x+2}{4x+8}=\frac{3x(x+2)+1(x+2)}{4(x+2)}=\frac{(3x+1)(x+2)}{4(x+2)}=$$ |
| $$19. \frac{18x+12}{15x+10}$$ | $$20. \frac{8x^{4}+4x^{2}}{10x^{5}}$$ | $$21. \frac{x^{2}+8x+12}{3x+6}$$ |
| $$22. \frac{5x^{2}+3x-8}{3x-3}$$ | $$23. \frac{5x-15}{-2x^{2}+6x}$$ | $$24. \frac{9x^{5}+27x^{3}}{18x^{4}}$$ |